ELECTROSTATICS

Types of charge

An amber rod rubbed with dry wool for attracts small pieces of paper. A glass rod rubbed with dry silk attracts small bits of paper. The rods are said to be electrified (to acquire charge) through the rubbing process.

Two amber rods rubbed with fur repel each other; similarly two glass rods rubbed with silk repel each other. An amber rod rubbed with fur attracts a glass rubbed with silk

The above observation show that:-

i. Unlike charges attract whereas like charges repel
ii. There exist two types of charges

The charge that appears on a glass rod rubbed with silk has been labeled positive and that which appears on an amber rod rubbed with fur is negative.

Conservation charge

The algebraic sum of the electric charges in any closed system remains constant. Uncharged objects contain equal amounts of negative and positive charge. When a glass rod is rubbed with silk, negative charge is transferred from the glass rod to the silk, leaving the glass rod with equal and opposite charge. For the closed system consisting of a glass rod and silk, the algebraic sum of the electric charge is constant.

Electrons and atoms

An atom consists of a positively charged, sense nucleus, consisting of protons and neutron. Reading about the nucleus are the electrons. The charge of the electron is negative and equal to $-1.6\times10^{-19}C$. It is the smallest charge, it is possible to obtain. The charge on the proton is $1.6\times10^{-19}C$.

The number of proton in the nucleus of an atom is called the atomic number and it is denoted by Z. Hence a neutral atom has positive charge equal to ^{+}Ze and positive charge equal to ^{-}Ze where $e=1.6\times10^{-19}C$.

The mass of an electron is $9.11\times10^{-3}kg$ and this is about $\frac{1}{1840}$ of the mass of the proton. The neutron has no charge; its mass is slightly greater than the mass of a proton.

Conductors, insulation and semi conductors

Atoms in solids are closely packed. The nuclei are separated by distances of 10^{-10}m. In conductors, most of the electrons are bound to the parent nuclei but about one electron per atom is free to wander through the lattice and can easily drift in one atom under an extreme electric field.

Insulators or di-electrics are materials in which all atomic electrons are bound to their potent nuclei. Electrons can be removed or added to an insulator only by expenditure of larger amounts of energy.

Semi conductors lie between conductors and insulators in their conduction properties. In semi conductors, a few electrons are free, the no of free electrons can be increased by heating.

conductors	semi conductors	insulators
Metals	Silicon, Germanium	Vacuum, plastics

Charging by rubbing

When two unlike di-electric are rubbed together, heat is produced. The thermal energy is sufficiently to cause removal of the weakly bound electrons in one material. The former is left with net positive charge while the latter acquires net negative charge of equal magnitude as the positive charge.

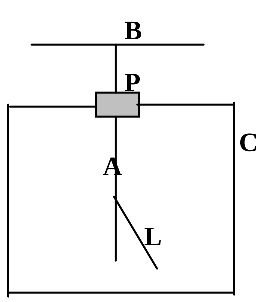
Charge conservation

The total electric charge on any object is on integral multiple of electrons since the charge is simply the algebraic sum of the charges of the elementary particles that make up the body. The charge on an elementary particle is either positive, zero or negative. Hence the charge scale equipments because the experiments involve a large number of electrons.

Charge detection

An electroscope can be used to detect charge. This consists of a metal rod A, to which a gold leaf or a very thin aluminum foil L is attached. The metal rod is fitted with a circular metal cup or disc B and is insulated from the metal casing C by means of a plug P.

The casing has glass or perpex windows through which the gold leaf may be observed.

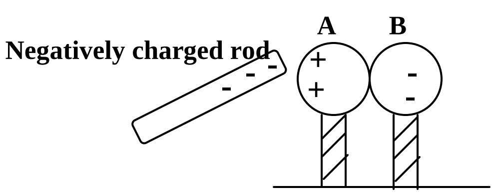


The metal casing screens the gold leaf from external influences.

Suppose the electroscope is given negative charge by touching the cup with an amber rod previously rubbed with fur. The gold leaf discharges. If a boxy X of unknown charge is brought near the cup and an increased divergence is observed, then the charge on X is negative. A decrease in divergence means that either X is neutral or it is positively charged. To distinguish between the two alternatives, the electroscope is discharged and then given positive charge X is then brought near the cap of the electroscope. An increase in divergence of the gold leaf implies that X is positively charged. A decrease in divergence means that X is neutral.

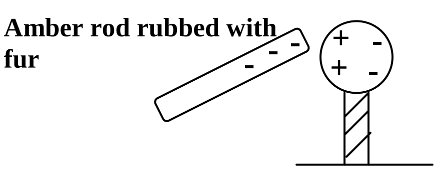
Electrostatic induction

Consider two identical metal spheres A and B supported by insulating stands suppose the spheres are brought into contact and a negatively charged rod is brought in the vicinity of A, positive charge is attracted towards the negatively charged rod and negative charge is repelled. If A and B are separated while the negatively charged rod still in place and the rod then removed tests show that A carries positive charge while B carried an equal amount of negative charge.

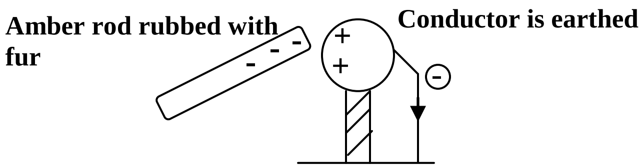


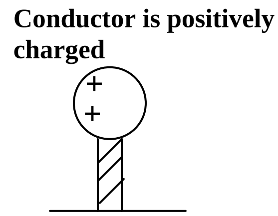
Charging by electrostatic induction

Suppose an amber rod R, rubbed with fur is brought near a conductor supported by an insulating stand as shown.



If the conductor is earthed when R is still in place, and then the rod removed, subsequent tests show that the conductor has acquired positive charge.



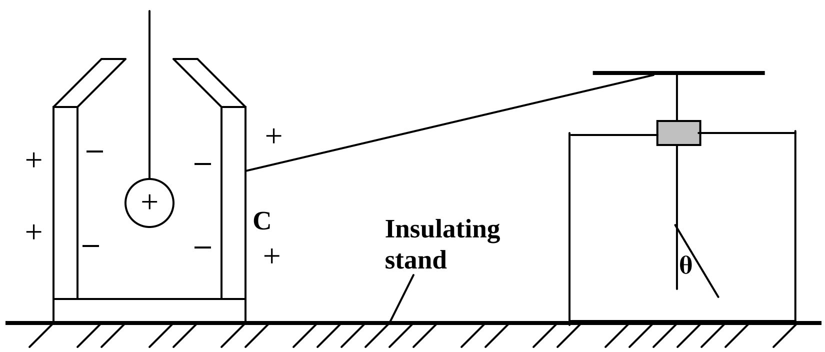


Explanation

Negative charge on the rod repels the loosely bound electron to the far side of the conducting sphere leaving charge on the near side. On earthing the sphere, electrons flow to the ground. When the earth is disconnected, the radial sphere left with positive charge. When the amber rod is withdrawn, the positive charge on the sphere distributes itself over the entire surface of the sphere.

Faraday's ice-pail experiment

The experiment reveals the manner in which charge is distributed on hollow conduction.



A metal sphere is suspended by an insulating thread is given positive charge. The sphere is then lowered into a metal can C connected to an uncharged electroscope as shown. The leaves of the electron are observed to diverge. The metal sphere is shiffed to various positions inside C but without touching C. the divergence θ on the electroscope is observed to remain the same.

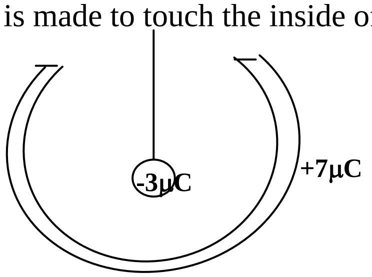
The sphere is withdrawn from C. the leaf of the electroscope collapses. The sphere is again lowered into C. the leaf of the electroscope is observed to diverge by the same amounts as before.

The sphere is then allowed to touch the inner surface of the can. The divergence θ is observed to remain constant. The sphere is then withdrawn and tested for charge. It is found to have lost all its charge, showing that there must have been an equal negative charge on the inner surface of the can which neutralized the positive charge on the sphere.

Hence, when a charged body is enclosed by a hollow conductor, it induces on the inside of the conductor equal but opposite charge. The total charge inside a hollow conductor is always zero, either there are equal and opposite charges on the inside walls and than the volume (as was the case before Sphere touched the can) or there is no charge at all. Any net charge on a hollow conductor resides on the outside surface of the conductor.

Example

A hollow spherical conductor carries a charge $7\,\mu C$ .A Pith having a charge of -3μC is introduced inside the spherical conductor in such a way that it does not touch the conductor. What happens when the pith ball is made to touch the inside of the conductor.



Positive charge +3μC is induced on the inside of the sphere. Negative charge of -3μC is induced on the outside of the sphere.

Hence the total charge on the outer surface is $^{+}(7-3)\mu C$ or $^{+}4\,\mu C$

The positive charge on the inside surface of the sphere is neutralized, when the ball touched the surface. This leaves only the positive charge of 4μC on the outside of the conductor.

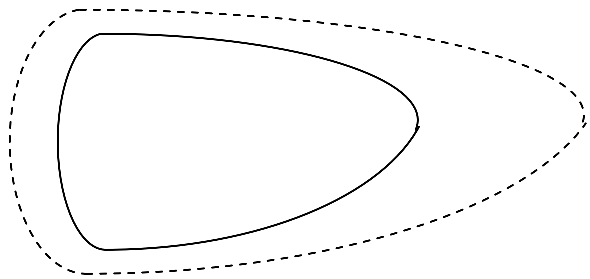
Distribution of charge on the conductor

Consider a pear-shaped positively charged conductor held on an insulator stand.

To investigate the distribution of charge on the conductor, a proof plane used. This is a small metal disc on an insulating handle. When the proof plane is placed on the surface of the conductor, a sample charge is acquired by the proof plane. The plane is then transformed into a hollow can on electroscope and the deflection noted. The proof plane and electroscope are discharged. Samples of charge are picked from different parts of the conductor and in each case the deflection on the electroscope noted.

In this way it is discovered that the surface density of charge on an unsymmetrical conductor is greatest where the curvature is greatest, that is, where the radius of curvature is least.

In the figure, the distance of the dashed curve from the surface of the conductor is proportional to the density of charge on the surface.



There is high density of charge at the pointed end of the conductor using proof planes, one can also investigate the way charge is distributed on a hollow conductor.

Corona discharge

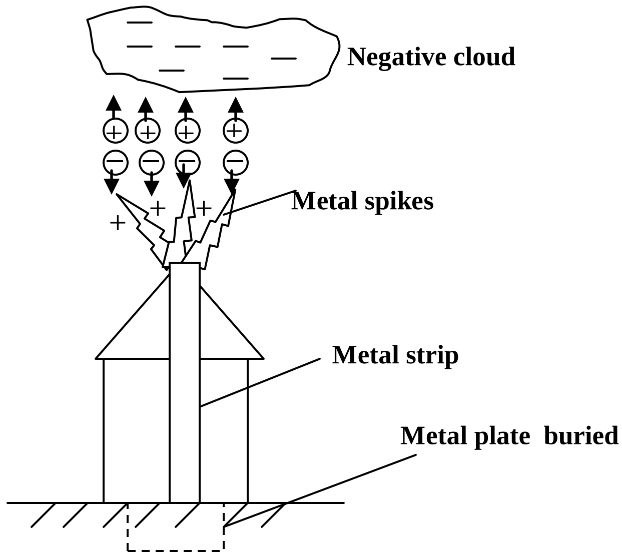
Consider a positively charged pointed particle in air. Because of high density of charge at pointed end, the electric field density there is very high. This causes air molecules near the sharp end to be ionized. Positive charges are repelled away from the sharp point while electrons are attracted into the conductor. Some of the charge on the conductor is neutralized. Thus the conductor loses some charge. It is though charge has leaked away. Below are two examples of corona discharge.

Applications of corona discharge

Lightening conductor

Lightening is rapid, high –current discharge between clouds or between a cloud and the ground. In the latter case, the large current when if passes through a building it can cause the building to burn down. Trees split up under the expansion of steam produced in the tree when the tree is struck by lightening. A lightening conductor is a sharp spiked conductor which is connected to a strip attached to a building and earthed.

Action: When a charged cloud passes over the conductor, opposite charge is induced on the conductor. A large electric field intensity exists at the pointed end which ionizes the air there.

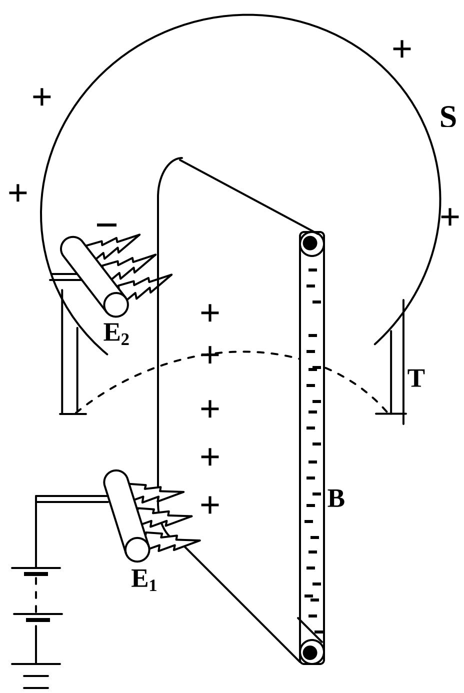


The ions having charge similar to that on the cloud are repelled to the ground

The positive ions are attracted to the cloud and neutralize some of the negative charge on the cloud.

The Van der Graff Generator

Main features



S - Metal hollow sphere

T -Insulting tube

B - Silk belt driven by a motor

E_1, E_2 - Metal electrodes of sharply pointed ends.

It consists of a large hollow metal sphere S supported on an insulated tube T, A silk belt inside the tube is driven by an electric motor past a sharply pointed metal electrode, E_1 held at an electric potential of about 10^4V relative to earth. As the belt moves upwards, it passes another sharply pointed metal electrode, E_2 connected on the inside of the hollow sphere.

Mode of operation

There is a high electric field intensity at the sharp ends E_1.

This ionizes the air there. The positive ions are repelled onto the belt. The positive charge is carried up by the belt towards the sphere it induces negative charge on the sharp ends E_2 and positive charge on the sphere to which the blunt end of E_2 is connected.

The high electric field held at the pointed ends of E_2 ionizes the air there, negative charges are repelled onto positive charge carried by the belt and neutralizes it before the belt passes over the upper pulley. This process of the belt charging up and discharging is repeated many times per second and each time the belt passes, the sphere S charges up positively until the electric potential is about 10^6V relative to earth. The electrical energy acquired comes from the work done by the motor to move the belt against a repulsion between positive charge on the sphere and positive charge on the belt.

Guiding questions

1. Describe an experiment to show that a charge resides only on the outside surface of the hollow conductor.
2. Describe the mechanism of charging by rubbing.
3. Describe what is meant by electrostatic screening.
4. Describe how a body can be charged but remains at zero potential.
5.

Coulombs law of electrostatics

The force between two charged bodies is directly proportional to the product of the magnitudes of their charges and inversely proportional to the square of the distance between them.

$F \propto Q_1 Q_2$

$F \propto \frac{1}{r^2}$

hence $F \propto \frac{Q_1 Q_2}{r^2}$, $F = k \frac{Q_1 Q_2}{r^2}$

Where $k = {}^{1}\!/_{4\pi\varepsilon}$

ε = permitivity of the medium in which the charges are placed.

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}$$

For free space or vacuum, $\varepsilon = \varepsilon_o$ (permittivity of free space)

$$\varepsilon_o = 8.85 \times 10^{-12}\, Fm^{-1}$$

hence $\quad k = \dfrac{1}{4\pi\varepsilon_o} = \dfrac{1}{4\pi \times 8.85 \times 10^{-12}} \simeq 9 \times 10^{9}\, mF^{-1}$

Example

Two identical point charges repel each other with a force of $1.0\text{x}10^{-5}$N. When the charges are moved 5mm further a part, the repulsive force reduces to $2.5\text{x}10^{-6}$N.

(i) How far apart were the charges originally?

(ii) What is the magnitude of the charge of each?

Solution

Let the original separation be x

$$F = \frac{Q^2}{4\pi\varepsilon_o r^2} = \frac{kQ^2}{r^2}$$

(i) $\therefore 10^{-5} = \dfrac{kQ^2}{x^2}$ ....(i)

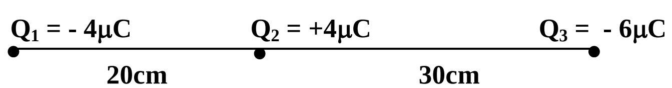
when separation is increased by 5mm

$$2.5 \times 10^{-6} = \frac{kQ^2}{(x + 0.005)^2} \text{.......... ....}(ii)$$

simplifying equation (i) and (ii) you get

$x = 0.005\ m$

2.



Find the force on **Q_3.**

solution

Let F_1 be the force induced on Q_3 due to Q_1

F_2 be the force induced on Q_3 due to Q_2

$$F_1 = \frac{kQ_1Q_2}{r_1^2}, r_1 = 0.5m$$

$$\frac{9\times10^{9}\times4\times10^{-6}\times6\times10^{-6}}{(0.5)^2}$$

$$= 0.864\ N\ (repulsive)$$

$$F_2 = \frac{kQ_2Q_3}{r_2^2}, r_2 = 0.3m$$

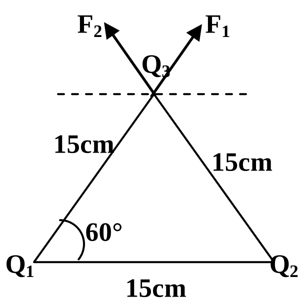
$$= \frac{9\times10^{9}\times4\times10^{-6}\times6\times10^{-6}}{(0.3)^2}$$

$$= 2.4\,N\ (\text{attractive})$$

$$\text{Net force} = (\mathrm{F}_2 - F_1) = 2.4 - 0.864$$

$$= 1.536\ N\ (\text{attractive})$$

3. Positive charges $Q_1 = 11\ \mu C, Q_2 = 15\ \mu C$ and $Q_3 = 20\ \mu C$ are located at the corners of an equilateral triangle of side 15cm. Calculate the magnitude and direction of the net force on



$$F_1 = \frac{kQ_1Q_3}{r_1^2} = \frac{9\times10^{9}\times11\times10^{-6}\times20\times10^{-6}}{(0.15)^2}$$

$$F_1 = 88\ N$$

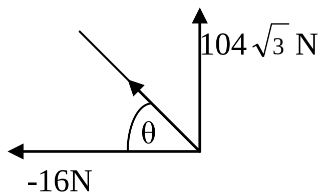
$$F_2 = \frac{kQ_2Q_3}{r_2^2} = \frac{9\times10^{9}\times15\times10^{-6}\times20\times10^{-6}}{(0.15)^2} = 120\ N$$

$$F_x = 88\cos 60 + -120\cos 60 = -16\ N$$

$$F_y = 88\sin 60 + 120\sin 60 = 104\sqrt{3}\,N$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(16^2 + \left(104\sqrt{3}\right)^2\right)} = 180.84\ N$$

Direction

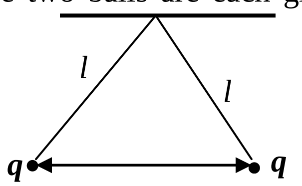


$$\tan\theta = \frac{104\sqrt{3}}{16}$$

$$\theta = 84.9^{o}$$

Questions

1. Two identical conducting balls of mass m are each suspended in air from a silk thread of length, l when the two balls are each given identical charges q, they more apart as shown in figure below.



If at equilibrium each thread makes a small angle **θ** with the vertical, show that the separation ***X***, is given by:

$$x = \left[\frac{q^2 l}{2\pi\varepsilon mg}\right]^{\frac{1}{3}} \text{where } \varepsilon \text{ is permitivity of air}$$

Electric field

An electric field is a region where an electric force is experienced.

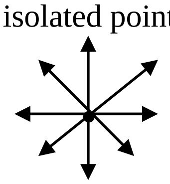
The direction of the electric field lines indicates the direction of the field. A higher density of the field lines indicate a string electric filed.

Properties of electric field lines

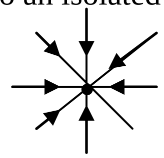
1. Each line of force start from the positively charged body and end at the negatively charged body
2. No two lines of force can cross each other
3. The density of lines of force at a point gives the direction of the electric field intensity at that point
4. Electric field lines are always normal to the surface of the conductor both when starting and ending on a conductor.

Electric field pattern

ii. Due to an isolated point positive charge



iii. Dues to an isolated positive charge



Students should sketch the following

iv. Two equal charges of same sign

v. Two unequal charges of opposite signs

vi. Two equal charges of opposite signs

vii. A point positive charge near a parallel plate

Electric field intensity

Electric field intensity of a point is the electric force exerted on one coulomb (1C) of a positive charge placed at that point i.e. force per unit charge.

The direction of the electric field intensity is always away from the positive charge and towards a negative charge.

Electric field intensity due to a charge

Consider a charge $\boldsymbol{Q^1}$ placed at a distance, $\boldsymbol{r}$ from a point positive charge, $\boldsymbol{Q}$

The force ***F*** *on* ***Q***^{1} is F = $\dfrac{QQ^{1}}{4\pi\varepsilon r_0^2}$

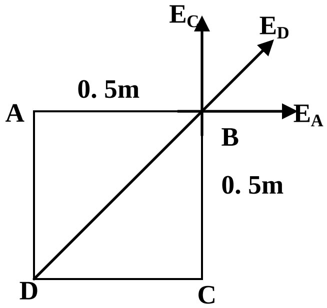
Force per unit charge, $E = \dfrac{F}{Q^{1}} = \dfrac{Q}{4\pi\varepsilon r_0^2}$

Hence electric field intensity, $E = \dfrac{Q}{4\pi\varepsilon r_0^2}$

Electric field intensity is a vector quantity.

Example

Calculate the electric field intensity at one corner of a square 0.5m if the outer three corners are occupied by point charges of magnitude $+8.24 \times 10^{-4} C$.



$$BD = \sqrt{\left(0.5^2 + 0.5^2\right)} = 0.707\ m$$

$$E_A = \frac{Q_c}{4\pi\varepsilon_o r_A^2} = \frac{9 \times 10^9 \times 8.4 \times 10^{-4}}{(0.5)^2} = 2.97 \times 10^7\ NC^{-1}$$

$$E_D = \frac{Q_D}{4\pi\varepsilon_o r_D^2} = \frac{9 \times 10^9 \times 8.4 \times 10^{-4}}{0.707^2} = 1.48 \times 10^7\ NC^{-1}$$

$$E_C = E_A = 2.97 \times 10^7\ NC^{-1}$$

$$\underset{\sim}{E} = \begin{pmatrix} 2.97 \times 10^7 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.48 \times 10^7 \cos 45 \\ 1.48 \times 10^7 \sin 45 \end{pmatrix} + \begin{pmatrix} 0 \\ 2.97 \times 10^7 \end{pmatrix}$$

$$\underset{\sim}{E} = \begin{pmatrix} 2.97 \times 10^7 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.05 \times 10^7 \\ 1.05 \times 10^7 \end{pmatrix} + \begin{pmatrix} 0 \\ 1.97 \times 10^7 \end{pmatrix}$$

$$\underset{\sim}{E} = \begin{pmatrix} 4.02 \times 10^7 \\ 4.02 \times 10^7 \end{pmatrix}$$

$$E = \sqrt{\left(4.02 \times 10^7\right)^2 + \left(4.02 \times 10^7\right)^2}$$

$$E = 5.67 \times 10^7\ NC^{-1}$$

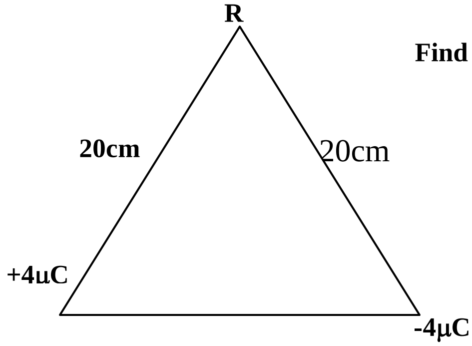
Direction

$$\tan \theta = \frac{4.02 \times 10^{7}}{4.02 \times 10^{7}} = 1$$

hence $\theta = 45^{0}$

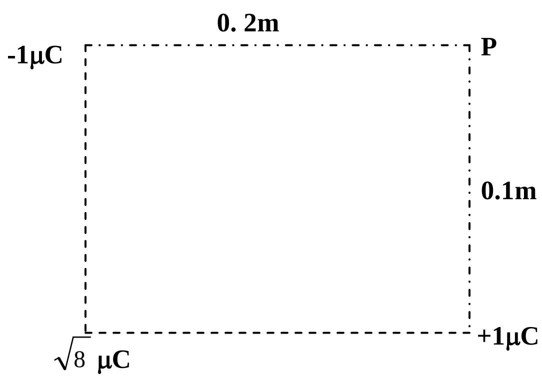
EXERCISE

1.



Find the intensity at R

2.

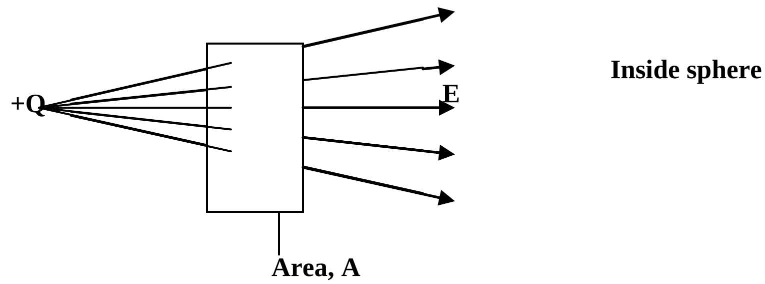


Find the intensity at P

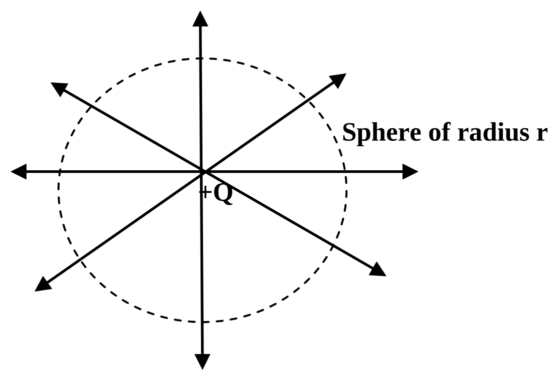
Electric flux

Electric field lines can be described by lines by force. The density of the lines increases near the charge where the field strength, E at a point can be represented by the number of lines per unit area or flux density, through the surface perpendicular to the lines of force at the point considered.

The *flux* through an area perpendicular to the lines of force *is the product of the electric field intensity and the area.*



Consider a sphere of radius r, drawn in space concentric with a point charge



Total normal flux = EA

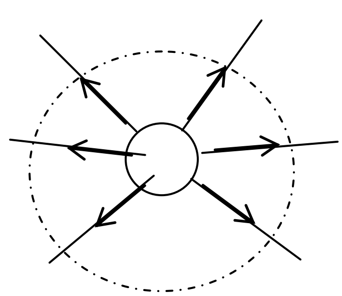
But $E = \frac{Q}{4\pi\varepsilon_o r^2}$ and $A = 4\pi r^2$

Hence flux = $\frac{Q}{4\pi\varepsilon_o r^2} \times 4\pi r^2 = \frac{Q}{\varepsilon_o}$

The total flux crossing normally any sphere outside and concentrically around a point charge is constant. In general, the total flux passing normally through any closed surface whatever its shape is always equal to $\frac{Q}{\varepsilon}$; where **Q** is the total charge endorsed by the surface and ε is the permittivity of medium. This relation is called ***Gauss' theorem***

Application of Gauss' theorem

Electric field intensity outside a charged sphere



Imaginary sphere of radius r

The flux across a spherical surface of radius ***r***, concentric with a small sphere carrying a charge ***Q*** is given by, flux = $\frac{Q}{\varepsilon}$

$$\Rightarrow \; E.A = Q/\varepsilon$$

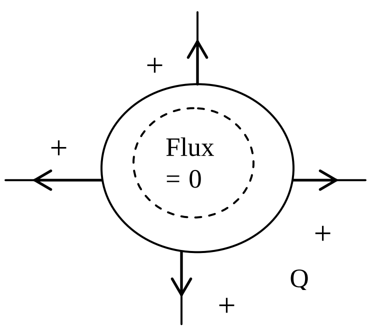
$$but A = 4\pi r^2$$

$$E.4\pi r^2 = Q/\varepsilon$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon r^2}}$$

The above expression is similar to that of a point charge. This means that outside a charged sphere, the field behaves as if all the charge on the sphere were concentrated at the centre.

Electric field intensity inside a charged surface

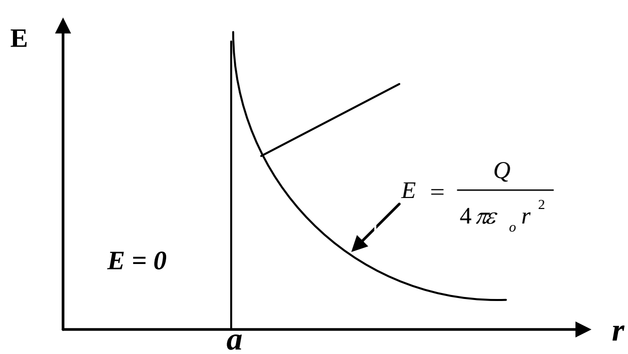


There is no charge inside the sphere hence flux = $\frac{Q}{\varepsilon} = \frac{0}{\varepsilon}$

$\therefore$ flux $= 0$; hence intensity, $E = 0$

Therefore inside, a charged body electric field intensity is zero.

Variation of E with distance r from the centre of sphere



Relation ship between electric field intensity and charge density

A charge per unit area of the surface of the conductor is called charge density,

$\sigma = Q/A$, where A = area

Consider a sphere of radius, r.

$$\sigma = \frac{Q}{A} = \frac{Q}{4\pi r^2} \quad \text{\_\_\_\_\_\_\_\_(1)}$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

From (1) and (2) $E = \dfrac{\sigma}{\varepsilon_o}$

Example

An isolated conducting spherical shell of radius **10cm** in a vacuum carries a charge of **+0.18μC**.

Calculate the electric field intensity at;

i. The surface of sphere
ii. A point placed outside the sphere at a distance of 5cm from the sphere surface
iii. A point 5cm from the centre of the sphere

Solution

i. Radius of the sphere

$$E = \frac{Q}{4\pi\varepsilon_o r^2} = \frac{0.18 \times 9 \times 10^9 \times 10^{-6}}{\left(10/100\right)^2} = 1.62 \times 10^5 NC^{-1}$$

ii. Radius of imaginary sphere, r = 15cm = 0.15cm

$$E = \frac{Q}{4\pi\varepsilon_o r^2} = \frac{0.18 \times 10^{-6} \times 9 \times 10^9}{0.15^2} = 7.2 \times 10^4 Nc^{-1}$$

iii. A point 5cm from centre of the sphere.

Inside a sphere, flux is 0, so electric field is zero

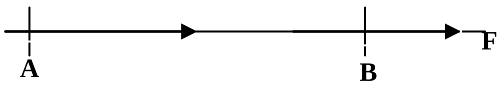
Electric potential

The work done to move one coulomb of a charge from infinity to that point

Potential is a scalar quantity. The unit of potential is the volt (V)

Potential difference (pd)

Consider two points A and B in an electric field



If one moves one coulomb of a positive charge from B to A, work is done against the field and this work done is called the potential difference. Therefore the p.d between two points is the *work done to move one coulomb of a positive charge from one point to another*. The unit of p.d is the volt (V)

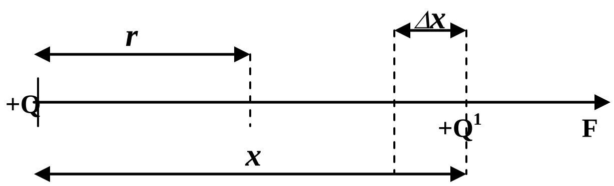
Definition of Volt: The p.d between two points is one volt if the work done to move one coulomb of a positive charge from one point to another is one joule.

$\therefore$ p.d $= \frac{\text{work done (w)}}{\text{charge (Q)}}$, $V = \frac{W}{Q}$ **Error! Bookmark not defined.**

1 volt = IJC^{-1}

Electric potential due to a point charge

Consider a point charge +Q^1 moved from infinity to a point which is a distance from a point charge +Q. If Q^1 is at a distance X from Q in free space, the force F on it F = $\frac{QQ^1}{4\pi\varepsilon_o x^2}$



The work done by an external force to move Q^1 by a distance **Δx** towards $+Q$,

$$\Delta W = -F\Delta x = -\frac{QQ1}{4\pi\varepsilon_o x^2}\Delta x$$

Hence; total work done to move ***+Q^1*** from infinity to a point which is a distance ***r*** from ***Q*** is

$$W = \int_{\infty}^{r} \frac{-QQ^1}{4\pi\varepsilon_o x^2}dx = \frac{QQ1}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$W = \frac{QQ1}{4\pi\varepsilon_o r}$$

But work done = potential energy. Hence Potential energy = $\frac{QQ_1}{4\pi\varepsilon_o r}$

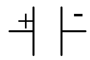
Hence electric potential V= $\frac{W}{Q_1}$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$$

CAPACITORS

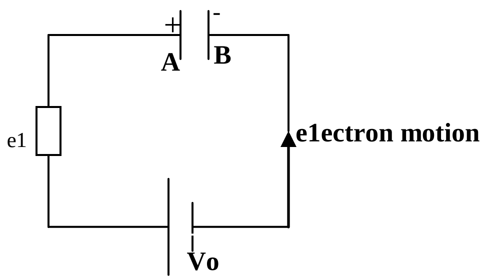
A device used for storing charge or electrical energy

Basically consists of two conduction, such as a pair of parallel metal plates separated by an insulator.

A symbol for a capacitor 

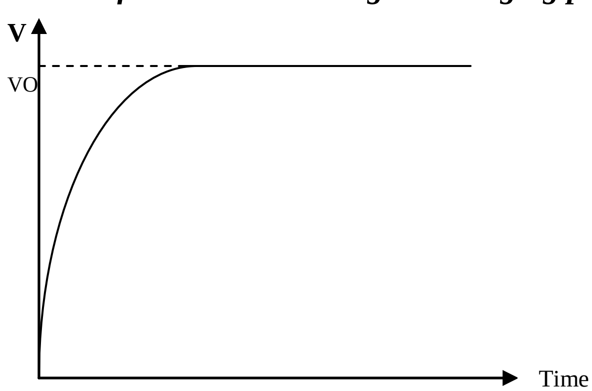
Charging and discharging of a capacitor

Electrons move from the negative terminal to plate B making it negatively charged. Electrons from plate B repel an equal number of electrons from plate A which then becomes positively.



In this case, the capacitor is said to be charging. The charging process continues until the p.d, V across the capacitor is equal to the p.d, Vo of the battery.

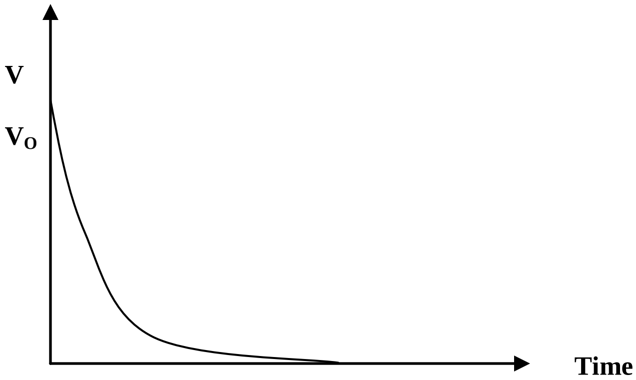
Variation of V with time during the charging process



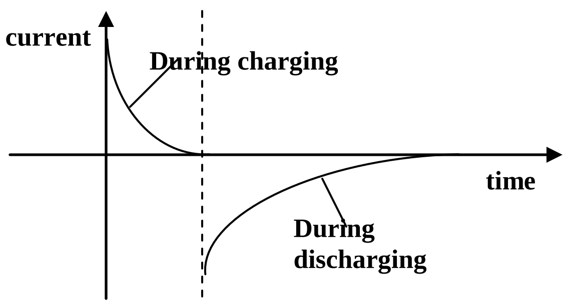
Generally, a capacitor is charged when a battery of p.d is connected to it.

When the plates of the capacitor are joined together, the capacitor becomes discharged (loses the charge on the plates)

Variation V with time during the discharging process



A graph of how current varies during charging and discharging process



Capacitance of a capacity (C)

Capacitance of a capacitor is the ratio of the magnitude of the charge on either plates to the p.d. between the plates of the capacitor

Capacitance, C $= \frac{charge(Q)}{p.d(V)}$

$$C = \frac{Q}{V}$$

Unit of C is the farad (F)

$IF - ICV^{-1}$ $\qquad$ $1pF = 10^{-12} F$

$I\mu F = 10^{-6} F$ $\qquad$ $InF = 10^{-9} F$

Capacitance of an isolated charged sphere

The potential V of a sphere of radius ***a***, in a medium of permittivity, ε, and having a charge Q is

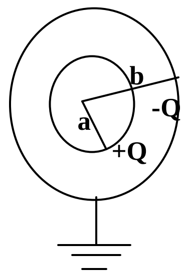
$$V = \frac{Q}{4\pi\varepsilon a} \qquad C = \frac{Q}{V} = Q \times \frac{4\pi\varepsilon a}{Q}$$

$$C = 4\pi\varepsilon a$$

for free space $C = 4\pi\varepsilon_o a$

Capacitance of concentric spheres

Consider two concentric sphere of radii a and b



Let Q^+ be the charge on the inner sphere and the outer sphere be earthed with air between them. The induced charge on the outer sphere is ^-Q. the potential

V_a of the inner sphere = Potential due to ^+Q + potential due to ^-Q.

$$V_a = \frac{+Q}{4\pi\varepsilon_o a} + \frac{-Q}{4\pi\varepsilon_o b}$$

$$Va = \frac{Q}{4\pi\varepsilon_o a} - \frac{Q}{4\pi\varepsilon_o b} = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\varepsilon_o} \times \frac{(b-a)}{ab}$$

$$Va = \frac{Q(b-a)}{4\pi\varepsilon_o ab}$$

Potential of outside sphere, $V_b = 0$ (since it is earthed, there is no charge)

Therefore the p.d between the inner and outer sphere $V = V_a - V_b$

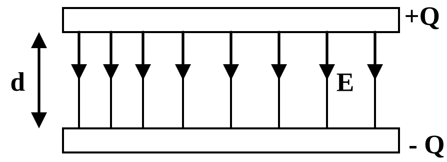
$$V = \frac{Q(b-a)}{4\pi\varepsilon_o ab} - 0 = \frac{Q(b-a)}{4\pi\varepsilon_o ab}$$

hence capacitance, $C = \frac{Q}{V} = \frac{4\pi\varepsilon_o ab}{(b-a)}$

$$C = \frac{4\pi\varepsilon_o ab}{(b-a)}$$

Capacitance of a parallel plate capacitor

Consider a capacitor with plates of common area A, separated by a medium of thickness d, and permittivity, ε. Of one plate has a charge +Q, the other with –Q.



Assuming that the field between the plates is uniform, the electric field intensity E is the same at all points and is given by $E = \frac{\sigma}{\varepsilon}$, where $\sigma = \frac{Q}{A}$ is charge density.

Hence $E = \frac{Q}{A\varepsilon}$. But $E = \frac{V}{d}$

Hence $\frac{V}{d} = \frac{Q}{A\varepsilon}$

$\frac{Q}{V} = \frac{\varepsilon A}{d}$

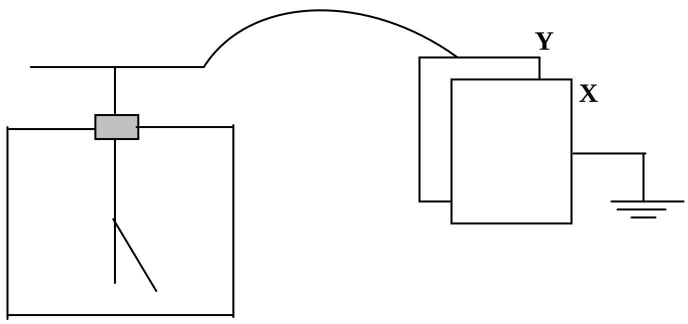
hence capacitance C = $\frac{Q}{V} = \frac{\varepsilon A}{d}$

$C = \frac{\varepsilon A}{d}$

If the medium between the plates is air, $C = \frac{\varepsilon_o A}{d}$

Experiment to show factors affecting the capacitance of a parallel plate capacitor.

(i) dependence of capacitance on separation, d, between plates.



Consider two plates X and Y with X earthed and Y connected to an electroscope.

Plates X and Y are set close to each other but not touching. Plate Y is given a charge Q by means of an electrophorus. The divergence of the leaf of the electroscope is a measure of the p.d between X and Y. Plate X is moved nearer to Y, without touching and the divergence of the leaf is observed to decrease meaning that the p.d between X and Y has decreased.

Hence capacitance, $C = \frac{Q}{V}$, increases. Hence decrease in separation between the plates of the capacitor increases with capacitance of the capacitor.

(ii) dependence of capacitance on area, A of overlap of plates.

Using the set in (i) plate Y is given a charge and the divergence of the leaf of electroscope noted. Plate X is then carefully displaced vertically downwards keeping separation, d, constant. The leaf of the electroscope is observed to increase meaning that the p.d between the plates has increased. Hence capacitance, $C = \frac{Q}{V}$, decreases.

By displacing plate X vertically downwards, the area of the plates decreases. Hence capacitance decreases with decrease of area.

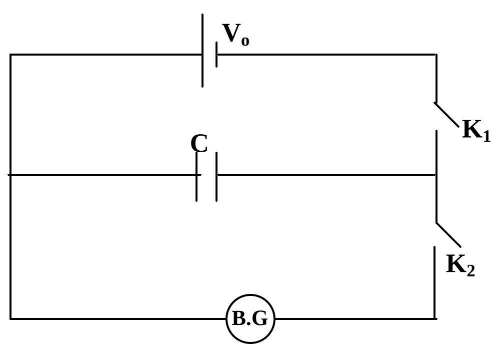
(Note: Area can be increased by joining another identical pair of metal plates to the previous pair of plates)

(iii) Dependence of capacitance on the nature of the medium between the plates.

Using the apparatus in step (i), the common area and separation are kept constant. Plate Y is again charged and the divergence of the leaf noted. A sheet of glass is placed between the plates X and Y. The leaf divergence is observed to decrease. This means that p.d between X and Y has decreased. Hence the capacitance $C = \frac{Q}{V}$, increases.

Therefore when a dielectric (insulator) is placed between plates of a capacitor , the capacitance of the capacitor increases.

Measurement of capacitance of a capacitor using a Ballistic galvanometer.



The capacitor of known capacitor C_1 is placed in position of C. Switch K_1 is closed for some time while K_2 is open. It is then opened.

In this case the capacitor C1 charges to p.d V_o, where its charge, $Q_1 = C_1V_o$.

K_2 is then closed and the deflection, θ_1, of the ballistic galvanometer.

Hence $Q_1 = k\theta_1$

This implies $C_1V_o = k\theta_1$ ....(i)

The capacitor of unknown capacitor C_2 replaces C_1. Switch K_1 is closed for some time while K_2 is open. It is then opened.

In this case the capacitor C_2 charges to p.d V_o, where its charge, $Q_2 = C_2V_o$.

K_2 is then closed and the deflection, θ_2, of the ballistic galvanometer.

Hence $Q_2 = k\theta_2$

This implies $C_2V_o = k\theta_2$ ....(ii)

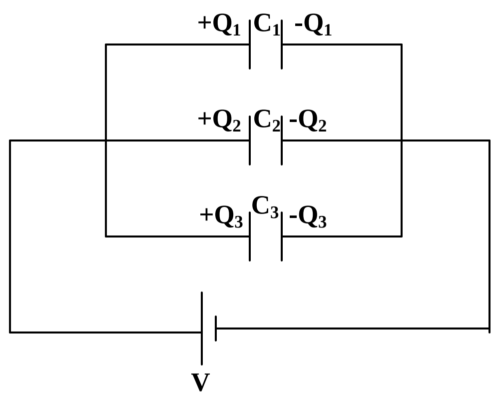
Hence equation (ii) divide by (i)

$$\frac{C_2}{C_1} = \frac{\theta_2}{\theta_1}$$

$$C_2 = \frac{\theta_2}{\theta_1} \times C_1$$

Arrangement of capacitors

(i) Parallel arrangement



The diagram above shows three capacitors connected in parallel to the same p.d, V.

The charges on the individual capacitors are respectively,

$Q_1 = C_1V$, $Q_2 = C_2V$, $Q_3 = C_3V$

total charge of the system, $Q = Q_1 + Q_2 + Q_3$

hence $Q = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$ ....(i)

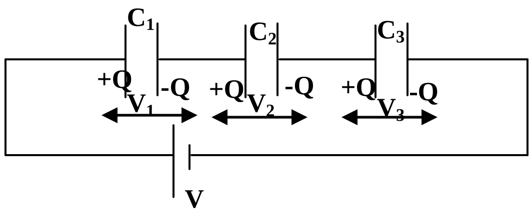
The system is equivalent to a single capacitor of capacitance $C = \frac{Q}{V}$

Hence $Q = CV$ ....(ii)

Substitute (ii) in (i) $CV = V(C_1 + C_2 + C_3)$

$C = (C_1 + C_2 + C_3)$

(ii) series arrangement



When a cell is connected in series, same charge appears on each capacitor plates. The p.d across the individual capacitors are therefore given by $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$, $V_3 = \frac{Q}{C_3}$

Hence total p.d, $V = V_1 + V_2 + V_3$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \text{……….(i)}$$

If C is the capacitance of the capacitance of capacitance equivalent to the system

Then $C = \frac{Q}{V}$

Hence $V = \frac{Q}{C}$ ……………..(ii)

Substitute (ii) into (i)

$$\frac{Q}{C} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

hence $\frac{1}{C} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

Case of two capacitors in parallel

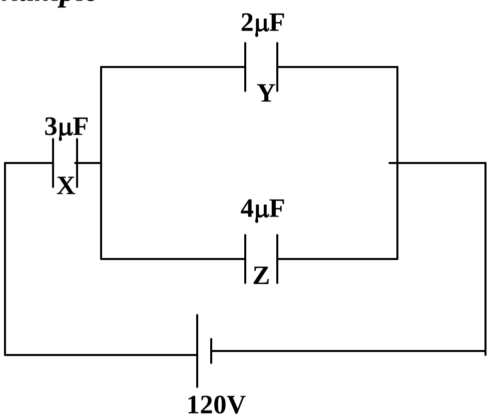
$$\frac{1}{C} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{1}{C} = \left(\frac{C_1 + C_2}{C_1 C_2}\right)$$

$$C = \left(\frac{C_1 \times C_2}{C_1 + C_2}\right)$$

$$C = \frac{product}{sum}$$

Example



Find (i) the p.d across each capacitor

(ii) the charge on each capacitor.

Y and Z are in parallel, hence their effective capacitance, $C_{YZ} = 2 + 4 = 6\mu F$

X and YZ are in series, hence their effective capacitance, $C_{XYZ} = \dfrac{6 \times 3}{6 + 3} = 2\mu F$

Total charge $Q = CV = 2 \times 10^{-6} \times 120 = 240 \times 10^{-6} C$

Hence $Q_x = 240x10^{-6}C$

Hence p.d across X, $V_x = \dfrac{Q_x}{C_x} = \dfrac{240 \times 10^{-6}}{3 \times 10^{-6}} = 80V$

Hence p.d across YZ, $V_{yz} = 120 - 80 = 40V$

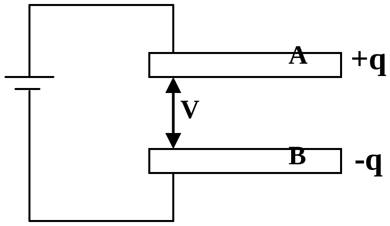
But $V_y = V_z = 40V$ (since they are parallel)

Therefore $Q_y = C_y V_y = 2 \times 10^{-6} \times 40 = 80 \times 10^{-6} C$

$Q_z = C_z V_z = 4 \times 10^{-6} \times 40 = 160 \times 10^{-6} C$

Energy stored in a capacitor

Consider a parallel plate capacitor of capacitance C being charged.



The charging process consists of transforming charge plate A tp plate B. Suppose that at some stage, during the charging process, the p.d between the plates is V when a small amount of charge $+\partial q$ is transformed from plate B to plate A. The charge on A increases to $q + \partial q$ and the p.d increases to $V+\partial V$. the work done to transfer charge $+\partial q$ from B to A is $\partial W = (V + \partial V)\partial q$

But $+\partial q$ is much smaller than q so even $\partial V <<$ V

Hence $\partial W = V\partial q + \partial V\partial q$

$\partial V\partial q \rightarrow 0$

hence $\partial W = V\partial q$

but $V = \frac{q}{C}$

$\partial W = \frac{q}{C}\partial q$

to charge the capacitor plate A from q = 0 to q = Q, the work done by the battery is

$$W = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C}$$

But Q = CV

Hence $W = \frac{CV^2}{2} = \frac{QV}{2} = \frac{Q^2}{2C}$

CURRENT ELECTRICITY

Conduction of electricity in metals

Metals have electrons that are loosely bound to the nuclei and are free to wander randomly through the metal from one point to the other. When the p.d is applied across the ends of the metal, an electric filed is set up. Electrons are accelerated by the field and so they gain kinetic energy. The accelerated electrons collide with atoms vibrating about a fixed mean position. They give out some of their energy to the atoms. The amplitude of vibrations of atoms is then increased and the temperature of the atoms rises.

Electrons are again accelerated by the field and again give up some of their energy on collision with the atoms of the metal. The overall acceleration of electrons is zero hence they acquire a constant average drift velocity, in the direction from the negative to the positive terminals of the battery. It is this drift of electrons that constitute an electric current.

In general, current flow in metals is due to movement of electrons (negative charge). In gases and electrolytes, both positive and negative charges are involved.

Unit of charge: the coulomb (C)

A coulomb is the quantity of electric charge carried past a given point in a circuit when a steady current of one ampere flows for one second.

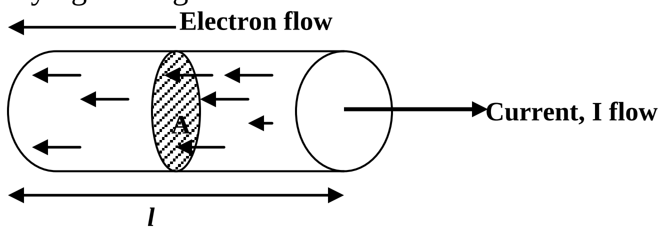
i.e. Charge, Q = current, I x time, t

Q = It

1C = 1As

Drift velocity

Consider a conductor of length *l*, cross sectional area A, having n electrons per unit volume, each carrying a charge e.



Volume of conductor = A*l*

Number of electrons in the conductor = nA*l*

Total charge, Q = nA*l*e

If *t* is the time taken by an electron to move through a distance *l*, then velocity, $v = \frac{l}{t}$

But I = $\frac{Q}{t} = \frac{nAle}{t}$

Hence $I = neVA$

Example

1. A current of 10A flows through a copper wire of area $1mm^2$. The number of free electrons per m^3 is 10^{29}. Find the drift velocity of the electron.

$$v = \frac{I}{nAe} = \frac{10}{10^{29} \times 1.6 \times 10^{-19} \times 1 \times 10^{-6}} = 6.25 \times 10^{-4} ms^{-1}$$

2. A metal wire contains $5x10^{22}$ electrons per cm^3 and has cross sectional area off $1mm^2$. If the electrons move along the wire with a mean drift velocity of $1mms^{-1}$, Calculate:

(i) current density

(ii) current in the wire.

Solution

(i) current density = $\frac{I}{A} = nev = 5 \times 10^{22} \times 10^{6} \times 10^{-3} \times 1.6 \times 10^{-19} = 8 \times 10^{6}\, Am^{-2}$

(ii) current = current density x Area, A = $8 \times 10^{6} \times 10^{-6} = 8\, A$

Ohm's Law

Under constant physical conditions, the potential difference across a conductor is proportional to the current through it.

$V \propto I$, hence $\frac{V}{I} = cons\tan t$

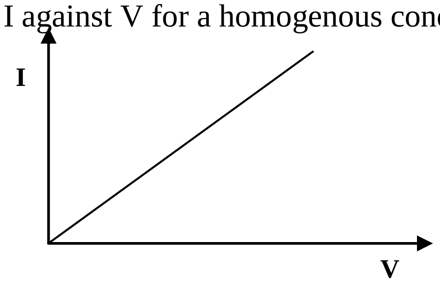
The constant is the resistance R of the conductor.

Hence $V = IR$

The unit of resistance is the ohm (Ω)

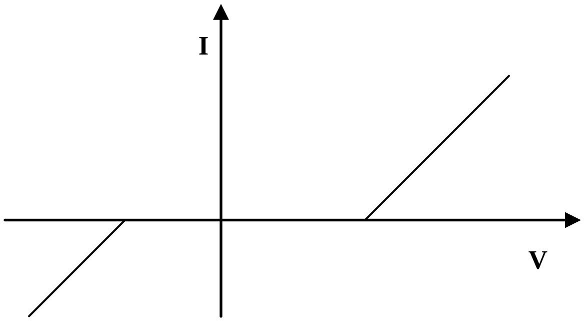
The ohm is the resistance of a conductor in which a current of one ampere flows when a p.d of one volt is applied across it.

A graph of I against V for a homogenous conductor.

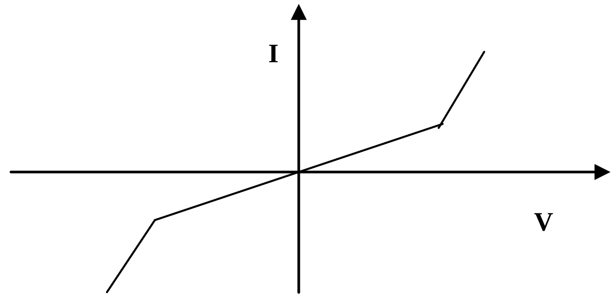


Graphs of I against V for non- Ohmic conductors

(i) Dilute sulphuric acid



(ii) Neon gas



(iii) electric bulb

(iv)

Resistivity (ρ)

The resistance, R of a metal wire is directly proportional to its length and inversely proportional to its cross sectional area, A

$R \propto l$ and $R \propto \frac{1}{A}$

Hence $R \propto \frac{l}{A}$

It follows that $R = \rho \frac{l}{A}$, where ρ is the resistivity of the material of wire

Consider a cube of the material of side 1m. Hence the area, A = $1m^2$.

From $R = \rho \frac{l}{A}$

Substitute l =1m, A = $1m^2$

$R = \rho$

Hence resistivity of a material can be defined as the resistance across opposite faces of a cube of a material of side 1m.

Unit of *ρ is ohm metre(Ωm)*

Example

A steady uniform current of 5mA flows along a metal cylinder of cross sectional area of $0.2mm^2$, length, 5m and resistivity $3x10^{-5}$Ωm. find the p.d across the ends of the cylinder.

$R = \rho \frac{l}{A} = \frac{3 \times 10^{-5} \times 5}{2 \times 10^{-7}} = 750\ \Omega$

Hence $V = IR = 5 \times 10^{-3} \times 750 = 3.75V$

Question

A p.d of 4.5V is applied to the ends of a 0.69m length of a wire of cross sectional area $6.6 \times 10^{-7} m^2$. Calculate the drift velocity of electrons across the wire. (ρ of wire is $4.3 \times 10^{-7} \Omega m$, number of electrons per m^3 is 10^{28} and electronic charge is 1.6×10^{-19}C)

Electromotive force, emf (ε) and internal resistance (r) of the cell

Emf of a source is the energy converted into electrical energy when 1C of charge passes through it.

Or it is the ratio of electrical power the source generates to the current which it delivers.

Internal resistance is the effective resistance of the source which accounts for the energy losses in it. Internal resistance behaves as if it is a resistor in series with the battery.

Hence

Energy supplied per coulomb by cell = (energy changed per coulomb by external circuit)

+(energy wasted per coulomb on internal resistance)

Hence $emf = V +$ 'lost p.d

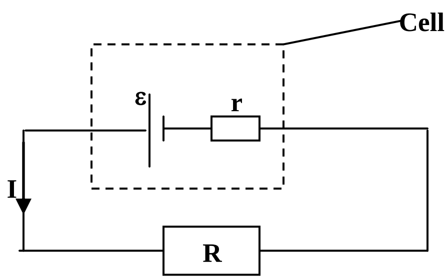
Where V is the p.d across the external circuit.

V is called terminal p.d and is the p.d across the cell in a closed circuit.

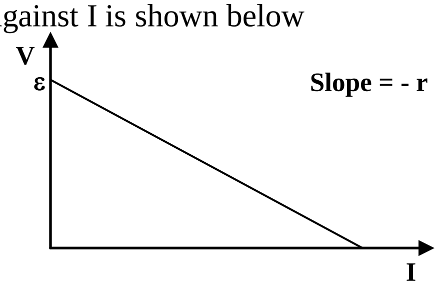
Hence ε = V + lost p.d

But lost p.d = I r, where I is the current flowing through the circuit.

Hence $\varepsilon = V + Ir$

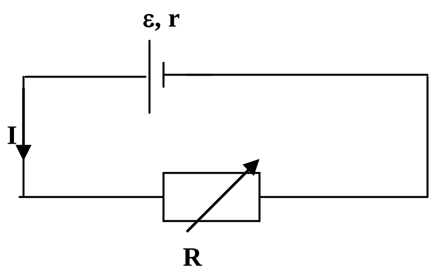


A graph of V against I is shown below



Also V = IR
Hence
$\varepsilon = IR + Ir = I(R + r)$

Power out put and efficiency



$$\varepsilon = I(R + r)$$

From $I = \dfrac{\varepsilon}{R + r}$

$$V = IR = \frac{\varepsilon R}{R + r}$$

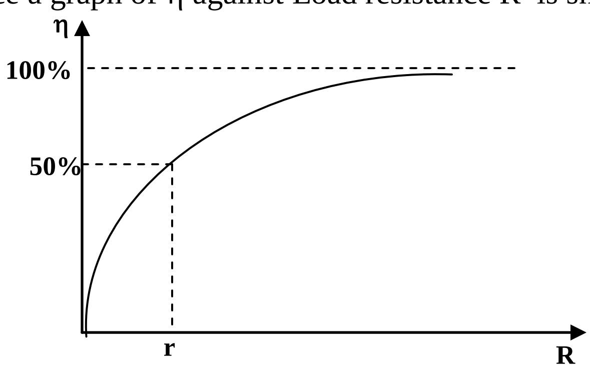
Eficiency,

$$\eta = \frac{Poweroutput}{powerinput} \times 100\ \% = \frac{VI}{\varepsilon I} \times 100\ \% = \frac{V}{\varepsilon} \times 100\ \%$$

$$= \left(\frac{\frac{ER}{R + r}}{E} \right) \times 100\ \% = \frac{R}{R + r}$$

If R = r, η = 50%; As $R \longrightarrow \infty$, $\eta \longrightarrow 100\%$

Hence a graph of η against Load resistance R is shown below:



Power out put, $P = VI = \dfrac{ER}{R + r} \times \dfrac{E}{R + r} = \dfrac{E^2 R}{(R + r)^2}$

At maximum power, P_m, $\dfrac{dP}{dR} = 0$

$$\frac{dP}{dR} = E^2 \frac{\left((R+r)^2 - 2R(R+r)\right)}{(R+r)^4} = 0$$ at P_m.

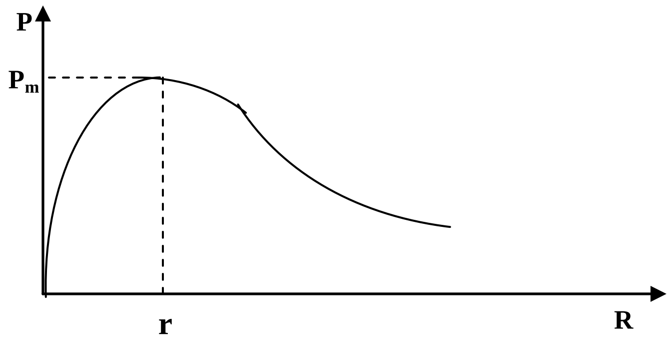
hence $R = r$

Power output, $P = \frac{E^2}{4r}$

As R tends to zero, P tends to zero

As R tends to ∞, P tends to zero.

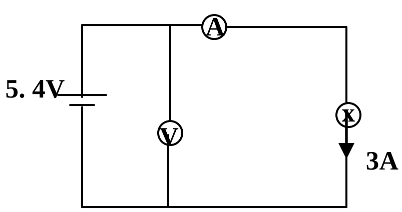
A graph of power out put P against load resistance R is shown below.



Exercise

1. When a 10Ω resistor is connected across the terminals of the cell of emf, E and internal resistance, r, a current of 0.1A flows through the resistor. If the 10Ω is replaced with a 3Ω resistor, the current increases to 0.24A. Find E and r.

2.



In the circuit above, V reads 4.8V. Calculate

(i) the internal resistance of the cell

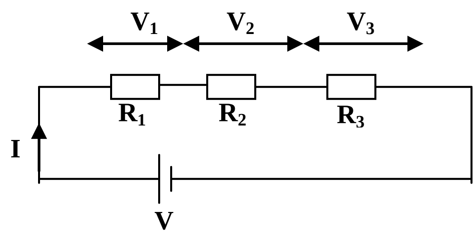
(ii) the energy transformed per second in the lamp

(iii) State any two assumptions made in calculations.

3. A voltmeter with resistance 20KΩ is connected across the power supply and gives a reading of 44V. Another voltmeter with a resistance of 50KΩ connected across the same supply gives a reading of 50V. Find the emf of the power supply.

Resistor Net works

(i) Series arrangement of resistors



In series

(i) same current flows through each resistor

(ii) total p.d V = sum of p.d across each resistor.

Hence $V = V_1 + V_2 + V_3$

Using ohm's law, $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$

Hence $V = I(R_1 + R_2 + R_3)$

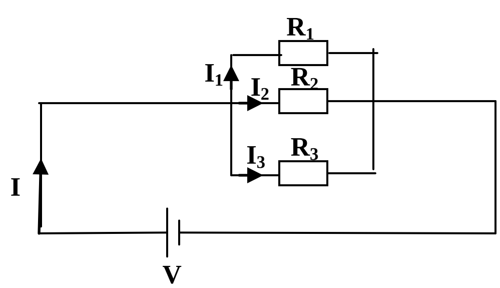
If R is the resistance of a single resistor representing the three resistors then,

V =IR

Hence $IR = I(R_1 + R_2 + R_3)$

$R = (R_1 + R_2 + R_3)$

(ii) Parallel arrangement of resistors



For parallel

(i) same p.d across each resistor

(ii) total current, I is equal to sum of Current through each resistor.

Hence $I = I_1 + I_2 + I_3$

Using ohm's law, $V = I_1R_1$, $V = I_2R_2$ and $V = I_3R_3$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

If R is the resistance of a single resistor representing the three resistors then,

V =IR

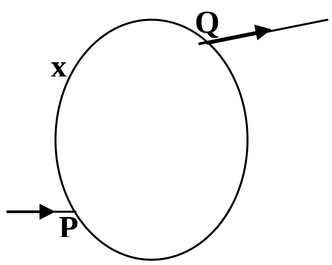
Hence $\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

Question

Should show that for two resistors in parallel, the effective resistance $R = \frac{R_1 \times R_2}{R_1 + R_2}$

Example

A wire of diameter ***d,*** length ***l*** and resistivity ***ρ*** forms a circular loop. Current enters and leaves at points **P** and **Q**.



Show that the resistance R of the wire is given by $R = \frac{4\rho x(l - x)}{\pi d^2 l}$

Let R_1 and R_2 be resistance of portion x and l-x of the wire respectively.

$$R_1 = \frac{\rho x}{A}, R_2 = \frac{\rho(l - x)}{A}$$

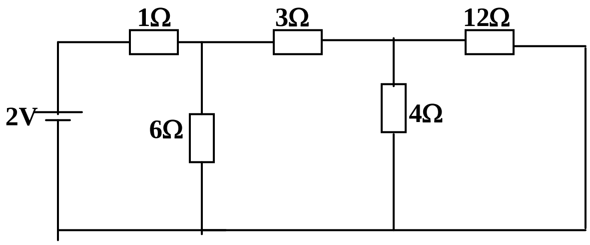
The two portions are in parallel, hence $R = \frac{R_1 \times R_2}{R_1 + R_2}$

$$R = \frac{\frac{\rho x}{A} \times \frac{\rho(l - x)}{A}}{\frac{\rho x}{A} + \frac{\rho(l - x)}{A}} = \frac{\rho x(l - x)}{Al}$$

But $A = \frac{\pi d^2}{4}$

Hence $R = \frac{4\rho x(l - x)}{\pi d^2 l}$

2.



Find the current supplied by the battery.

4 Ωand 12Ω resistors are parallel, their effective resistance is $R_1 = \frac{4 \times 12}{4 + 12} = 3\Omega$

R_1 and 3Ω resistors are in series, their effective resistance is $R_2 = R_1 + 3 = 3 + 3 = 6\Omega$

R_2 and 6Ω resistors are in parallel, their effective resistance is $R_3 = \frac{6 \times 6}{6 + 6} = 3\Omega$

R_3 and 1Ω resistors are in series, their effective resistance is $R = R_3 + 1 = 3 + 1 = 4\Omega$

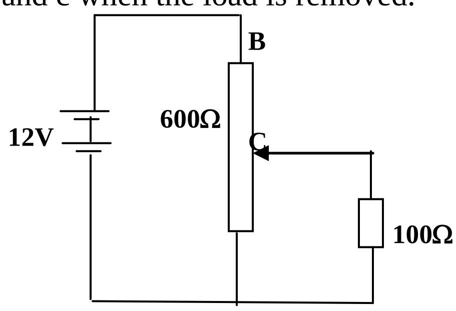
Hence effective resistance of the whole circuit is R = 4Ω

Current flowing $I = \frac{V}{R} = \frac{2}{4} = 0.5A$

3. a 12V battery is connected across a potential divider of resistance 600Ω as shown below. If the load of 100Ω is connected across the terminals a and c when the slider is half way up the divider, find:

(i) p.d across the load

(ii) p.d across a and c when the load is removed.



Effective resistance

$R = \frac{300 \times 100}{300 + 100} + 300 = 375\ \Omega$

current supplied by the battery, $I = \frac{V}{R} = \frac{12}{375} = 0.032\ A$

hence current through parallel combination of resistors = 0.032A

p.d across parallel combination of resistors, $V = IR' = 0.032 \times 75 = 2.4V$

where $\left(75 = \frac{300 \times 100}{300 + 100}\right)$

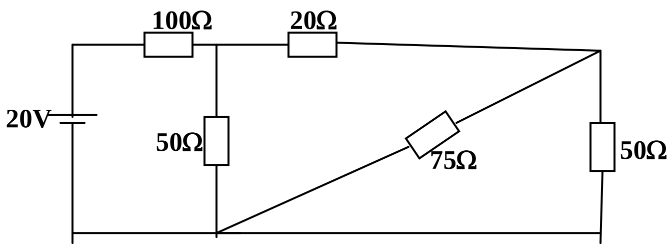
Hence the p.d across the load is 2.4V.

(iii) when the load is removed $I = \frac{12}{600} = 0.02\ A$

Hence p.d across AC = $0.02 \times 300 = 6V$

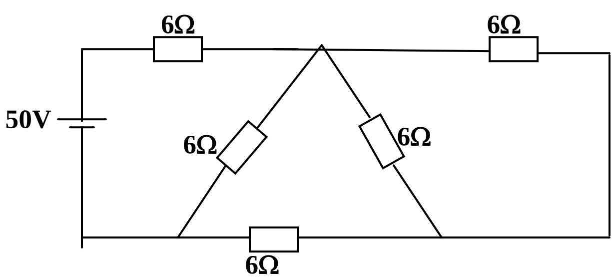
Exercise

1.



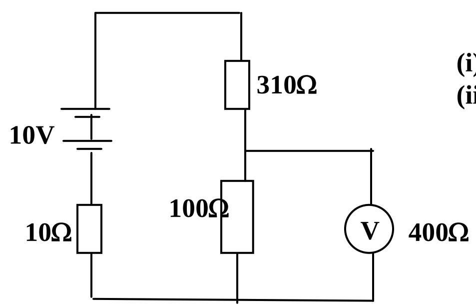
Find (i) the current supplied by the dc source.
(ii) power dissipated in the 20Ω resistor.

2.



Given that the internal resistance of the battery in the circuit is 0.4Ω, find
(i) the effective resistance in the circuit
(ii) power dissipated in the battery.

3.



(i) Find the reading of the voltmeter
(ii) Calculate the power dissipated in the 10Ω resistor

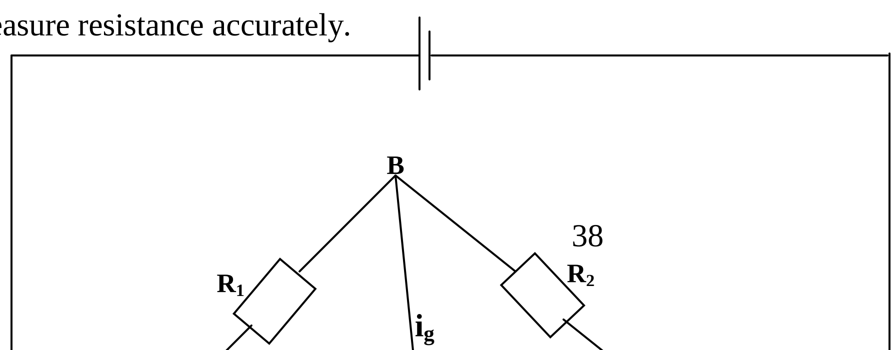
4. A resistor of 500Ω and one of 200Ω are placed in series with a 6V supply. What will be the reading on a voltmeter of internal resistance 2000Ω when placed across

(i) the 5000Ω resistor

(ii) 2000Ω resistor.

Wheatstone bridge

A Wheatstone bridge circuit is an arrangement of four resistors R_1, R_2, R_3 and R_4. It used to measure resistance accurately.



G is centre zero galvanometer

R_1 is unknown resistor

R_2 is standard resistor (known)

R_3 and R_4 are variable resistors where resistance can be read e.g. resistance box.

As R_3 and R_4 are varied, the potential at B and D change.

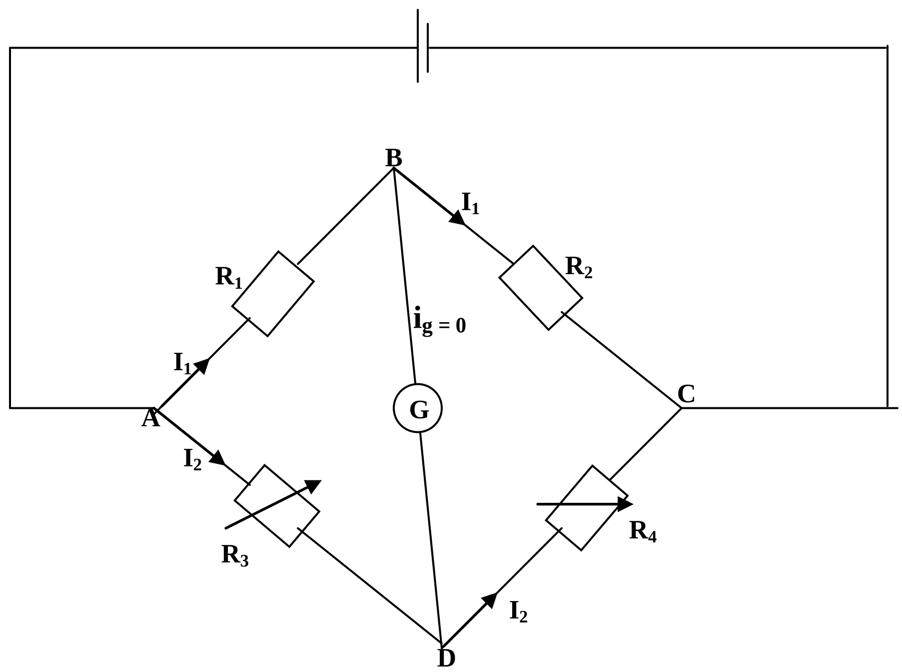
(i) If the potential at B, V_B is greater than that at D, V_D, then current i_g flows from B to D giving a deflection on one side of the galvanometer.

(ii) If $V_D > V_B$, i_g flows from D to B, galvanometer flows in the opposite direction.

(iii) If $V_D = V_B$, $i_g = 0$, galvanometer indicates zero deflection. In this case the Wheat stone bridge is said to be *balanced*.

Therefore to *balance* the bridge, R_3 and R_4 are varied until the galvanometer indicates zero deflection.

Conditions for the Wheatstone bridge circuit to balance



At balance, $i_g = 0$, hence $V_D = V_B$

Hence

$$V_{AD} = V_{AB}$$
$$I_2 R_3 = I_1 R_1 \text{..........} \quad ...(i)$$

Similarly,

$$V_{DC} = V_{BC}$$
$$I_2 R_4 = I_1 R_2 \text{..........} \quad ...(ii)$$

equation (i) divide by equation (ii)

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

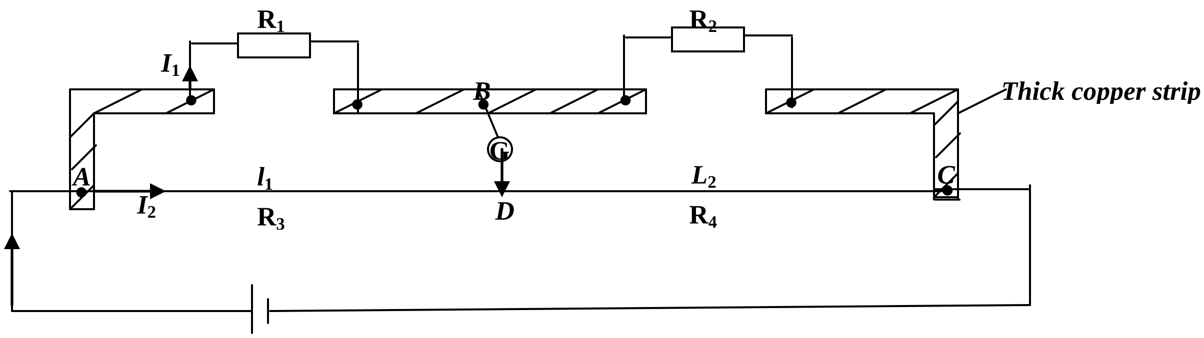
The above equation is the condition for a Wheatstone bridge to balance.

Questions

In a Wheatstone bridge, the ratio arms R_1 and R_2 are approximately equal. When $R_3 = 500\Omega$, the bridge is balanced. On interchanging R_1 and R_2, the value of R_3 for balancing is 505Ω. Find the value of R_4 and the ratio $R_1 : R_2$. (*502.5Ω, 1:1.005*)

Simple metre bridge

It is a form of a Wheatstone bridge with a resistance wire of uniform cross section area mounted on a metre rule. It is used to measure resistance, resistivity and temperature coefficient of resistance.



Condition for a metre bridge to balance.

At balance, $i_g = 0$, hence $V_D = V_B$

Hence

$$V_{AD} = V_{AB}$$
$$I_2 R_3 = I_1 R_1 \ldots\ldots\ldots\ldots(i)$$

Similarly,

$$V_{DC} = V_{BC}$$
$$I_2 R_4 = I_1 R_2 \ldots\ldots\ldots\ldots(ii)$$

equation (i) divide by equation (ii)

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

But $R_3 = \frac{\rho l_1}{A}$, where A is the cross sectional area of the uniform wire of the metre bridge, ρ is the resistivity of the material of wire, l_1 is the balance length of the metre bridge from the left hand side.

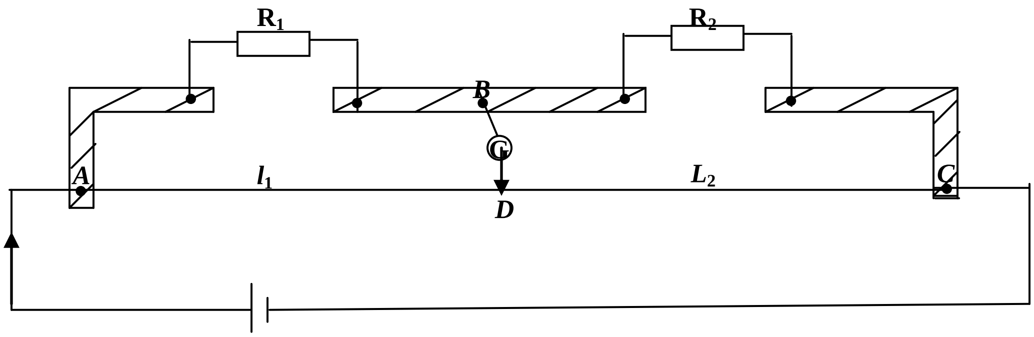
$R_4 = \frac{\rho l_2}{A}$, where l_2 is the balance length of the metre bridge from the right hand side.

Hence $\frac{R_1}{R_2} = \frac{l_1}{l_2}$

The above equation is the condition for a metre bridge to balance.

If the length of the wire is 100cm then $l_2 = (100 - l_1)$cm

End errors or end corrections



Owing to imperfect electrical contacts at A and C, contacts have small resistance which affect the accuracy of the results. But the effect on the accuracy becomes less significant if the balance point is near the 50cm mark.

For accuracy, the contact resistances have to be considered. The contact resistances at A and C are equivalent to extra lengths e_1 and e_2 of the slide wire. e_1 and e_2 are called end corrections or end errors.

Hence end errors have to be added to the balance lengths to account for the resistances at the contacts at the ends of the slide wire.

At balance $\frac{R_1}{R_2} = \frac{l_1 + e_1}{l_2 + e_2}$ ....(i)

When R_1 and R_2 are interchanged

$$\frac{R_1}{R_2} = \frac{l_1^{'} + e_1}{l_2^{'} + e_2} \text{.......... ..}(ii)$$

where $l_1^{'} and l_2^{'}$ are new balance lengths from the left hand side and right hand side respectively.

Solve equations (i) and (ii) simultaneously to obtain e_1 and e_2.

Example

When resistors of 3Ω and 5Ω are connected in the LHD and RHG of a metre bridge respectively, a balance point is obtained at 37.4cm from LHS. When the resistors are interchanged, the balance point is 62.8cm from the LHS. The resistance of the slide wire is 10Ω. calculate the end corrections and resistance of the contacts at LHS and RHS.

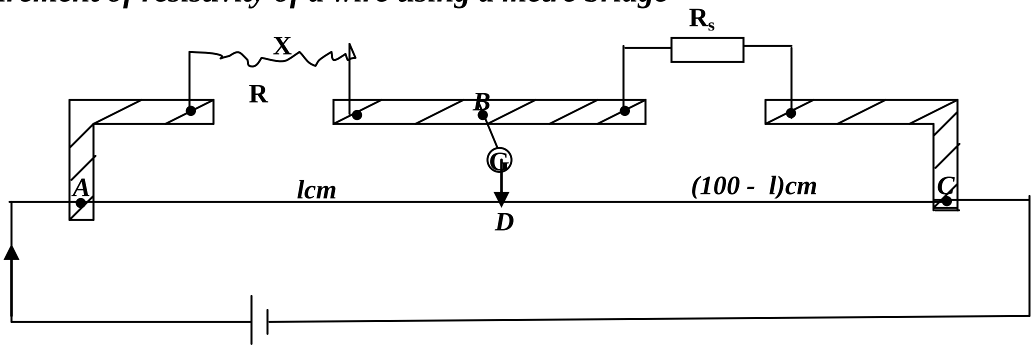
$$\frac{3}{5} = \frac{37.4 + e_1}{62.6 + e_2} \text{.......... .....}(i)$$

$$\frac{5}{3} = \frac{62.8 + e_1}{37.2 + e_2} \text{.......... .....}(ii)$$

Solve equations (i) and (ii) simultaneously you obtain e_1 = 0.7cm and e_2 = 0.9cm.

Resistances from zero end r_1=0.07Ω, on the right end r_2 = 0.09Ω.

Measurement of resistivity of a wire using a metre bridge



A specimen wire of known length x is connected in the left hand gap of a metre bridge with a standard resistor, R_s in the right hand gap of the bridge. The slider is moved along AC until the galvanometer indicates zero deflection. At this point the metre bridge is balanced. The balance length l from the left hand side of the metre bridge is measured. The experiment is repeated for different values of *x* and the corresponding balance lengths l are measured and recorded.

Theory of experiment

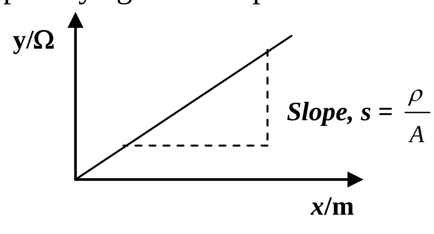
At balance, $\frac{R}{R_s} = \frac{l}{100 - l}$

$$R = \frac{R_s l}{100 - l} = y$$

But $R = \rho \frac{x}{A}$

Hence $y = \rho \frac{x}{A}$

A graph of y against *x* is plotted



The slope of the graph is determined and is equal to s = $\frac{\rho}{A} = \frac{\rho}{\frac{\pi d^2}{4}}$

Hence resistivity, $\rho = s \times \frac{\pi d^2}{4}$. d is the diameter of the wire which can be measured by a micrometer screw gauge.

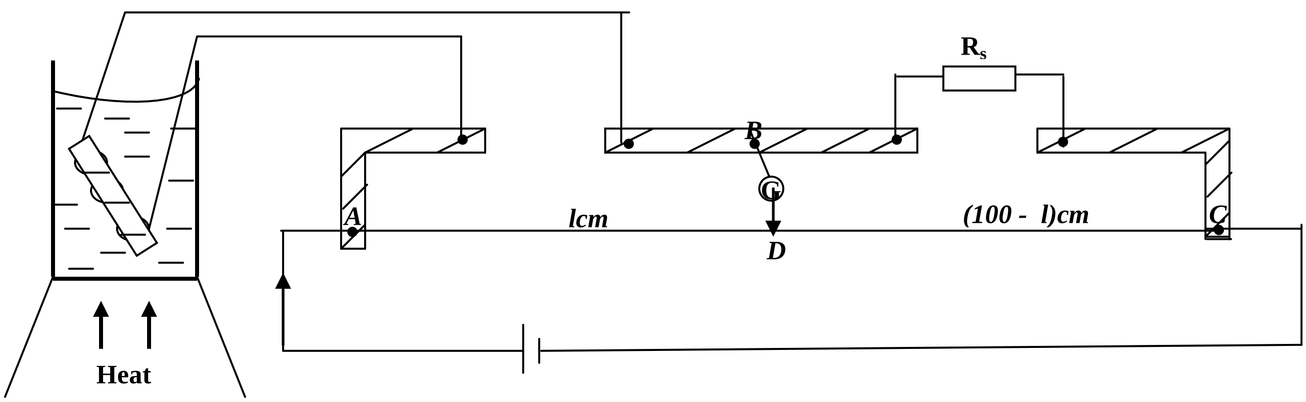
Temperature coefficient (α) of a metal wire

The mean temperature coefficient of a material can be defined as the fractional increase in resistance at 0°C for every °C increase in temperature.

$\alpha = \frac{R_\theta - R_0}{R_0 \theta}$, R_0 is resistance at 0°C, R_θ is the resistance at temperature θ°C.

hence $R_\theta = R_0 (1 + \alpha\theta)$ For semi conductors, resistance decreases as temperature increases hence α is negative.

Experiment to determine temperature coefficient of resistance using the metre bridge

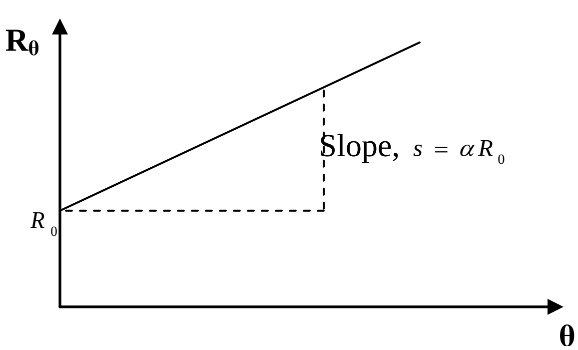


The specimen wire is made into a coil and immersed in water basin whose temperature θ°C is varied and measured with a thermometer placed in the water. The coil is connected to the left hand gap of the bridge and a standard resistor R_s to the right hand gap.

At each temperature, θ°C, the balance length l from the left hand side id measured and recorded.

The resistance R of the coil is calculated using the formula $R_\theta = \frac{R_s l}{100 - l}$

A graph of R_θ against θ is plotted.



Temperature Coefficient of resistance, $\alpha = \frac{s}{R_0} = \frac{slope}{\text{int } ercept}$

Examples

1. When a coil x is connected across the Left hand gap of a metre bridge and heated to a temperature of 30°C, the balance point is found to be 51.5°C from the left hand side of the slide wire. when the temperature is raised to 100°C, the balance point is 54.6cm from the left hand side. Find the temperature coefficient of resistance of x.

$$R_{30} = \frac{R_s 51.5}{100 - 51.5} = 1.06 R_s$$

$$R_{100} = \frac{R_s 54.6}{100 - 54.6} = 1.203 R_s$$

But $R_{30} = R_0(1 + 30\alpha)$.......... (i)

$$R_{100} = R_0(1 + 100\alpha) \text{..........} \quad \text{......}(ii)$$

$$\frac{R_{30}}{R_{100}} = \frac{1.06 R_s}{1.203 R_s} = \frac{1 + 30\alpha}{1 + 100\alpha}$$

Hence $\alpha = 2.01 x 10^{-3} K^{-1}$

Note:

The Wheatstone bridge can measure accurately resistances from 1Ω to 10^6Ω . it cannot be used to measure resistances less than 1Ω because the contact resistances become comparable to the test resistances. The bridge cannot be used to measure accurately the resistance above 10^6Ω because the galvanometer becomes insensitive.

Exercise

1. Two resistance coils P and Q are placed in the gaps of a metre bridge. A balance point is found when the movable contact touches the bridge wire at a distance of 35.5cm from the end joined to end P. When the coil Q is shunted with a resistance of 10Ω, the balance point is moved through a distance of 15.5cm. Find the values of the resistances P and Q.

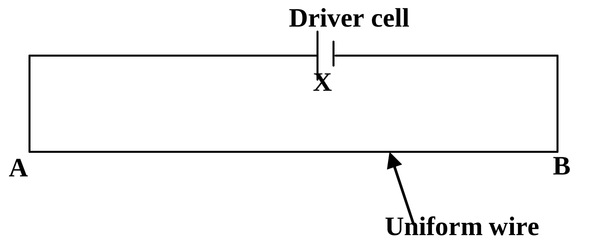
2. In a metre bridge when a resistance in left gap is 2Ω and unknown resistance in right gap, the balance point is obtained from the zero end at 40cm on the bridge wire. On shunting the unknown resistance with 2Ω, find the shift of the balance point on the bridge wire. (22.5cm)

3. With a certain resistance in the left gap of a slide wire, the balancing point is obtained when a resistance of 10Ω is taken out from the resistance box. On increasing the resistance from the resistance box by 12.5Ω, the balancing point shifts by 20cm. Find the value of unknown resistance. (15Ω)

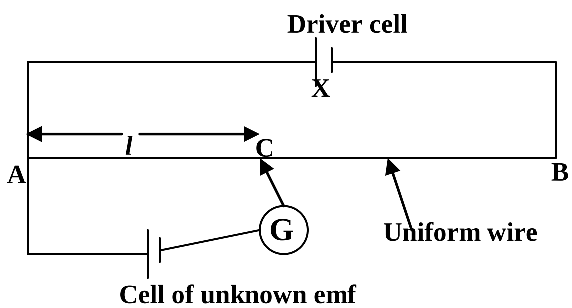
Potentiometers

Potentiometers are used to measure p.d. As a result of measuring resistance and current can also be obtained.

Potentiometer consists of a uniform wire of length 1m long mounted on a metre rule. A driver cell is used to maintain a current through the wire.



The unknown p.d to be determined is connected in opposition to the driver cell.



The slide (jockey) is moved along a slide wire until a point is reached where the galvanometer does not deflect. At this point the potentiometer is said to be balanced.

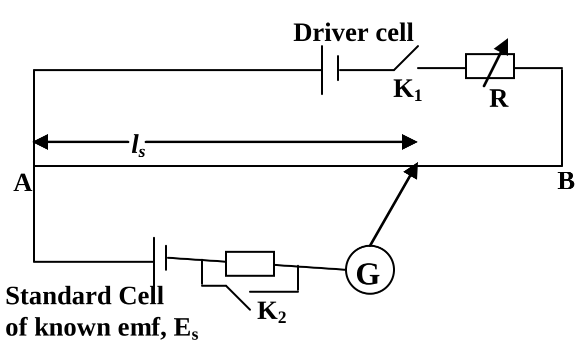
At balance, the p.d across the balance length is equal to the p.d of unknown cell. Hence emf of cell is equal to the pd across the balance length. But p.d across balance length is proportional to balance length.

i.e. $V_{AC} \propto l$ where *l* is balance length.

$V_{AC} = kl$, k is constant called p.d per unit length.

If *l* is in cm, then k = p.d/cm

Calibration/ standardization of a potentiometer.



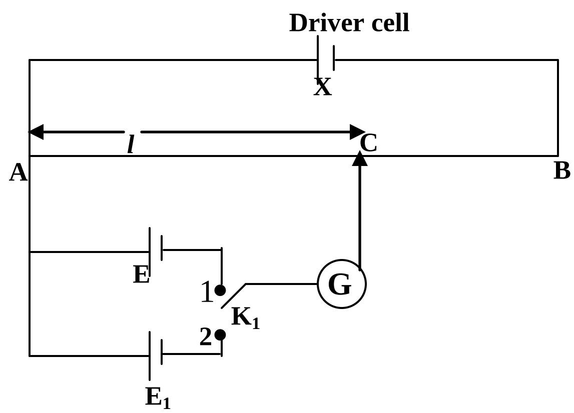
Switch K_1 is closed and the circuit checked for a two way deflection by moving the slide contact towards A and B. G should deflect in opposite direction.

The sliding contact is moved along AB until the galvanometer shows no deflection. R is adjusted if necessary so that the balance point is beyond the 50cm mark.

Switch K2 is closed and the point located accurately using the full sensitivity of the galvanometer. The balance length $\boldsymbol{l_s}$ is measured and recorded.

The calibration constant $K = \frac{E_s}{l_s}$

***Measuring emf/comparing emf using a potentiometer*.**



Switch K_1 is connected to position 1 and the sliding contact(jockey) is moved along the wire AB until the galvanometer indicates zero deflection. The corresponding balance length l is measured.

At balance, $E = kl$.......... ..(i)

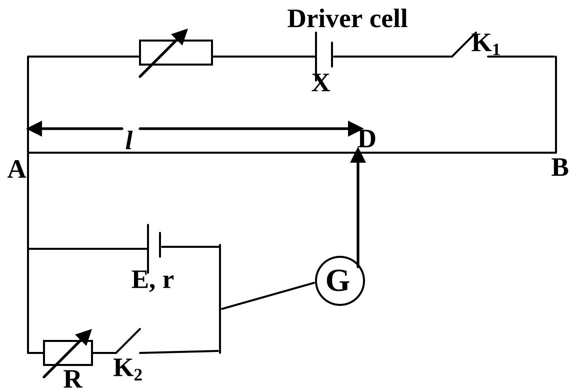
K_1 is then connected to position 2, the jockey is moved along the slide wire AB until the galvanometer indicates zero deflection, and the corresponding balance length l_1 is measured.

At balance $E_1 = kl_1$.......... ..(ii)

Equation (i) divide it by equation (ii)

$$\frac{E}{E_1} = \frac{l}{l_1}$$

Measuring internal resistance using potentiometer

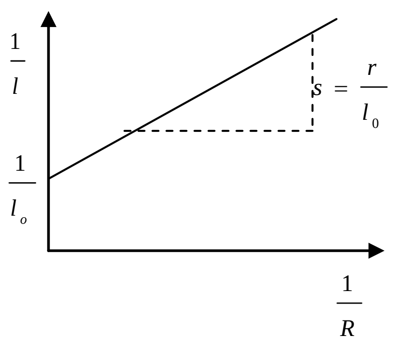


Switch K_1 is closed. A jockey is moved along wire AB until the galvanometer indicates zero deflection. The balance length l_s is measured ad recorded.

Switch K_2 with is then closed with R set to suitable value of resistance. The balance point where the galvanometer indicates zero deflection is obtained. The corresponding balance length l is measured and recorded. The experiment is repeated for different settings of R and each time the corresponding balance length l is measured.

The results are recorded in a suitable table including values of $\frac{1}{l}$ and $\frac{1}{R}$.

A graph of $\frac{1}{l}$ against $\frac{1}{R}$ is plotted.



The slope of the graph, s is determined and is $s = \frac{r}{l_o}$

But $\frac{1}{l_o}$ is the intercept on the $\frac{1}{l}$ axis

Therefore internal resistance $r = \frac{slope}{\text{int } ercept}$

Theory of experiment

When K_1 is closed and K_2 open, $E = kl_s$ ……………(i)

When both K_1 and K_2 closed $V = kl$ ……………….*(ii)*

Equation (i) divide by (ii)

$$\frac{E}{V} = \frac{l_s}{l}$$

But $V = IR$

$$I = \frac{E}{R + r}$$

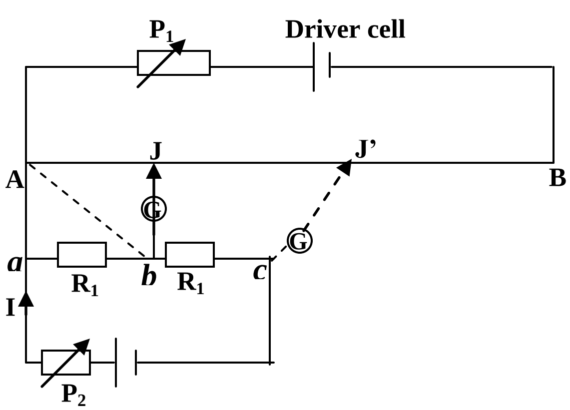
Hence $V = \dfrac{ER}{R + r}$

Therefore $\dfrac{E}{V} = \dfrac{R + r}{R}$

Hence $\dfrac{1}{l} = \dfrac{1}{l_s} + \left(\dfrac{r}{l_s}\right)\dfrac{1}{R}$

Hence a graph of $\dfrac{1}{l}$ against $\dfrac{1}{R}$ gives a slope, $s = \dfrac{r}{l_o}$

Comparison of resistance using a potentiometer



The two resistors to be compared are connected in series so that the same current flows through them. With the galvanometer at ***a*** and ***b***, the balance length AJ = l_1 is measured and recorded.

Hence $IR_1 = kl_1$.......... ...(i)

Connections at ***a*** and ***b*** are removed and replaced by those at ***b*** and ***c***(dotted lines).

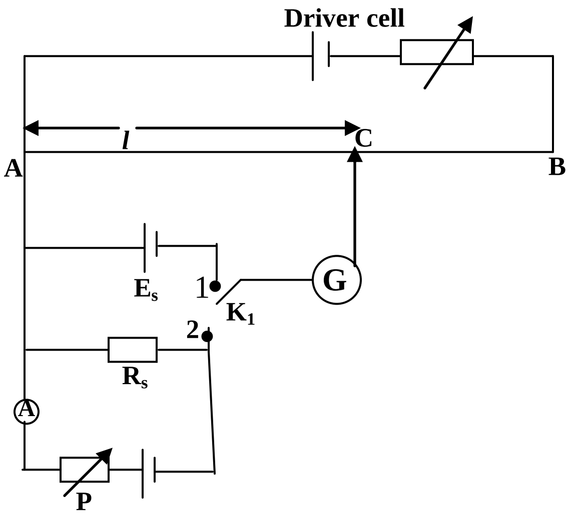
The balance length AJ' = l_2 is measured and recorded.

Hence $IR_2 = kl_2$.......... ...(ii)

Equation (i) divide by (ii)

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

Calibration of Ammeter using potentiometer



Switch K_1 is connected to position 1. A balance point is located where the galvanometer indicates zero deflection. The balance length l_s is measured and recorded.

P is adjusted so that the ammeter records the smallest current. K_1 is then connected to position 2 and the balance length l is obtained and recorded. The experiment is repeated for different adjustments of P and hence for different readings of the ammeter. Balance length l is determined in each case and the results tabulated including $I' = \frac{E_s}{l_s} \times \frac{l}{R_s}$

A graph of I' against is plotted and constitutes a calibration curve.

Theory of experiment

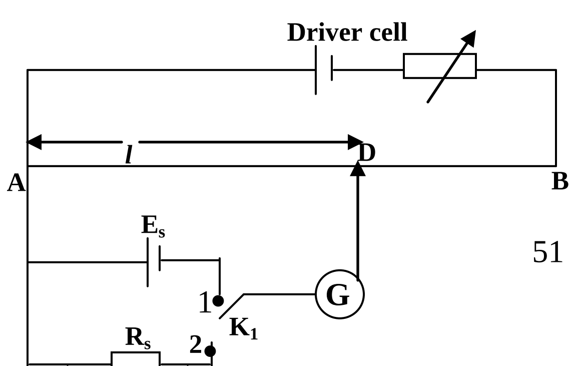
When K_1 is in position 1, $E_s = kl_s$

Hence $k = \frac{E_s}{l_s}$

When K_1 is in position 2, $I'R_s = kl$

$$I' = \frac{kl}{R_s} = \frac{E_s l}{R_s l_s}$$

Calibration of Voltmeter



Switch K_1 is connected to position 1. A balance point is located where the galvanometer indicates zero deflection. The balance length l_s is measured and recorded.

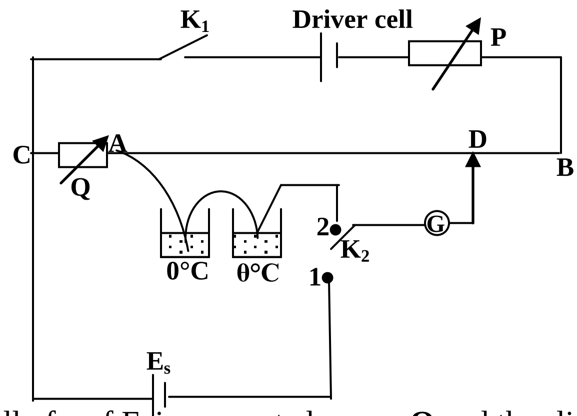
Hence p.d per cm, $k = \frac{E_s}{l_s}$, where Es is the emf of the standard cell

P is adjusted so that the voltmeter records the smallest p.d it possible to read. K_1 is then connected to position 2 and a point is located where the galvanometer register zero current. The balance length l is obtained and recorded. The experiment is repeated for different adjustments of P and hence for different readings, V_r of the voltmeter. Balance length l is determined. The results are tabulated including values of $V_a = kl$

A graph of V_a against V_r is plotted and it constitutes the calibration curve of the voltmeter.

Measurement of small emf (e.g. emf of a thermocouple)

Thermocouple emf's are of the order o several mV. Hence it is very small to cause any considerable balance length. In order to measure emf of a thermocouple, a slide wire AB of the potentiometer is connected in series with high resistance, R.



The standard cell of emf E_s is connected across **Q** and the slide wire. K_1 is closed and k2 is connected to position P. P and Q are adjusted keeping (P+Q) constant. The position is found on AB for which the galvanometer shows no deflection. For accuracy AD >20cm. The balance length l_s is measured and recorded.

While K_1 is closed, K_2 is connected to position 2 and the point on AB when the galvanometer registers zero current is found. The balance length *l* is measured.

E is found using the formula $E = \frac{E_s rl}{Q + rl_s}$ where r is the resistance per cm of the slide wire.

Theory of experiment

Current through the wire AB $i_p = \frac{V_o}{P + Q + rL}$ where L is the length of the wire.

Hence p.d per cm, $k = i_p r = \frac{V_o r}{P + Q + rL}$

When K_2 is in position 1, $E_s = i_p Q + kl_s = \frac{V_o}{P + Q + rL}(Q + rl_s) \ldots\ldots\ldots\ldots\ldots(i)$

When K_2 is in position 2, $E = kl$

Where E is the emf of the thermocouple.

$$E = \frac{V_o rl}{P + Q + rL} \ldots\ldots\ldots\ldots\ldots (ii)$$

equation (ii) divide by (i)

$$\frac{E}{E_s} = \frac{rl}{Q + rl_s}$$

Hence $E = \frac{rlE_s}{Q + rl_s}$

Advantages of potentiometer over a moving coil voltmeter

(i) The potentiometer is more accurate because it does not draw current from the circuit whose p.d it is meant to measure. The potentiometer can be considered to be a voltmeter with an infinitely high resistance which is ideal voltmeter.

(ii) The potentiometer method is a null method. The accuracy of the potentiometer does not depend on the accuracy of the galvanometer but only on its sensitivity. The accuracy of the result is not affected by the fault accuracy of the galvanometer.

Disadvantages of potentiometer over a moving coil voltmeter

(i) It does not give direct reading

(ii) It requires a skilled person

(iii) It is slow in operation.

Examples

1) The emf of a battery A is balanced by a length of 75cm on a potentiometer wire. The emf of the standard cell, 1.02V is balanced by a length of 50cm.

(i) What is the emf of A.

Using $\frac{E}{E_1} = \frac{l}{l_1}$

$$E = \frac{l}{l_1} E_1 = \frac{75}{50} \times 1.02 = 1.53\, V$$

(ii) Calculate the new balance length if A has internal resistance of 2Ω and a resistor of 8Ω is joined to the terminals.

$$\frac{E}{V} = \frac{l}{l_1}$$

$$\frac{1.53}{V} = \frac{75}{l_1}$$

but $V = IR = \frac{ER}{R + r} = \frac{1.53 \times 8}{8 + 2} = 1.224\, V$

$$\frac{1.53}{1.224} = \frac{75}{l_1}$$

Hence $l_1 = 60$cm

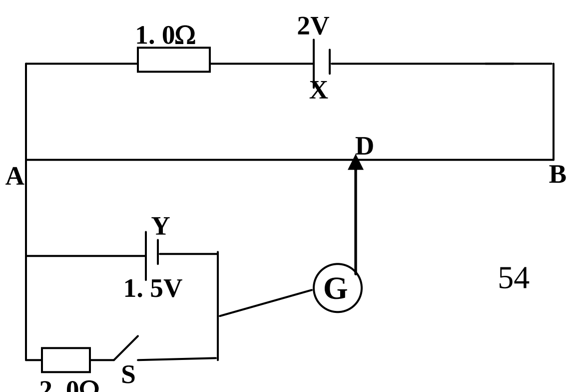
2. A dry cell gives a balance length of 84.8cm on a potentiometer wire. When a resistor of resistance 15Ω is connected across the terminals of the cell, a balance length of 75cm is obtained. Find the internal resistance of the cell.

$$\frac{E}{V} = \frac{l}{l_1} = \frac{84.8}{75}$$

$$V = \frac{E \times 75}{84.8} = \frac{ER}{R + r} = \frac{15}{15 + r}$$

$$r = 1.96\, \Omega$$

3.



AB is a uniform wire of length 1m and resistance 4Ω. X is battery of emf and negligible internal resistance 2V. Y has emf 1. 5V.

(i) Find the balance length of AD when the switch S is open.

(ii) If the balance length is 75cm, when the switch S is closed, find the internal resistance of Y.

(i) $E_y = kl$

$k = i_p r$

where $r = \dfrac{4}{100}\,\Omega\,cm^{-1}$

and $i_p = \dfrac{2}{1+4} = \dfrac{2}{5}A$

Hence $k = \dfrac{2}{5} \times \dfrac{4}{100} = \dfrac{8}{500} Vcm^{-1}$

Therefore

$E_y = kl$

$l = \dfrac{E_y}{k} = \dfrac{1.5}{8} \times 500 = 93.75\,cm$

(ii) when S is closed

$\dfrac{E_y}{V} = \dfrac{l}{l_1} = \dfrac{93.75}{75}$

$V = \dfrac{E_y \times 75}{93.75} = \dfrac{1.5 \times 75}{93.75} = 1.2V$

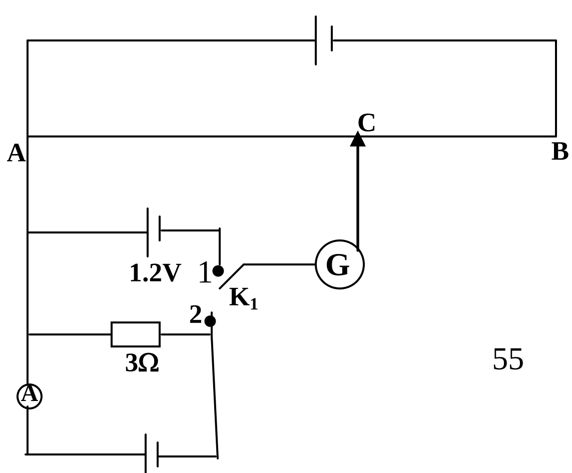
but $V = \dfrac{E_y R}{R+r} = \dfrac{1.5 \times 2}{2+r}$

hence $1.2 = \dfrac{3}{2+r}$

$r = 0.5\Omega$

4. In the circuit below, when K_1 is connected to position 1, the balance length AC =30.2cm.

When K_1 is connected to position 2, the balance length AC = 26.8cm and the ammeter reading is 0.4A. Find the percentage error in the ammeter reading.



When K_1 is in position 1,

$$1.2 = k \times 30.2 \text{..........} .(i)$$

When K1 is in position 2

$$V = k \times 26.8 \text{..........} .(ii)$$

$$V = \frac{1.2}{30.2} \times 26.8 = 1.065\text{V}$$

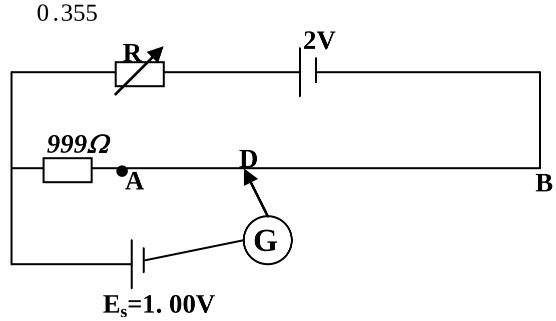
But V = IR =3I

Hence $I = \frac{1.065}{3} = 0.355\ A$

Error = 0.4 – 0.355 =0.045A

$$\%\ error = \frac{0.045}{0.355} \times 100 = 12.7\%$$

5.



The slide wire has length 100cm and resistance 10Ω. The galvanometer shows no deflection when AD = 10cm. Find

(i) the current flowing in the driver cell

(ii) the value of R.

(iii) the emf of thermocouple which balanced by a length of 60cm of the slide AB.

(i)

$$E_s = i_p \times 999 + kl_s$$

$$k = i_p \times \frac{10}{100} = \frac{i_p}{10}$$

$$E_s = i_p \times 999 + \frac{i_p \times 10}{10} = 1000\ i_p$$

but E_s =1.00V

Hence
$$1000\, i_p = 1$$
$$i_p = \frac{1}{1000} A$$

(ii)
$$i_p = \frac{2}{999 + 10 + R}$$
$$\frac{1}{1000} = \frac{2}{999 + 10 + R}$$

$R = 991\ \Omega$

(iii)

emf of thermocouple, $E = kl = \frac{i_p}{10} \times 60 = \frac{1}{1000 \times 10} \times 60 = \frac{6}{6000} = 6\, mV$

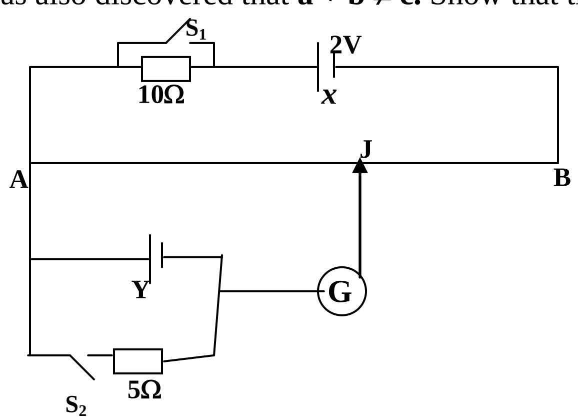
Exercise

1. A 1Ω resistor is in series with an ammeter m in a circuit. The p.d across the resistor is balanced by a length of 60cm on a potentiometer wire. A standard cell of emf 1.02V is balanced by a length of 50cm. If m reads 1.1A, what is the error in the reading? (0.124A)

2. A potentiometer wire of length 1m and resistance 1Ω is used to measure an emf of the order mV. A battery of emf 2V and negligible internal resistance is used as a driver cell. Calculate the resistance to be in series with potentiometer so as to obtain a potential drop of 5mV across the wire. (399Ω)

3. In a potentiometer, a cell of emf **x** gave a balance length of **a** cm and another cell of emf **y** gave a balance length of **b** cm. When the cells are connected in series, a balance length of **c** cm was obtained. It was also discovered that **a + b ≠ c.** Show that the true ratio $\frac{x}{y} = \frac{c - b}{c - a}$.

4.



In the circuit above, ***x*** has negligible internal resistance and length AB is 100cm and resistance of AB is 50Ω.when S_1 and S_2 is open, the balance length AJ = 90cm. When S_2 is closed and S_1 open, the balance length AJ = 75cm. Find

(i) the emf of cell y (1.5V)

(ii) internal resistance of cell y. (1Ω)

(iii) the balance length when S_1 and S_2 are closed. (62.5cm)