

P425/2
APPLIED MATHEMATICS
Paper 2
3 hrs.

STANDARD HIGH SCHOOL ZZANA

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

*Attempt **all** the eight questions in section **A** and any **five** questions from section **B**.*

*Any additional question (s) answered will **not** be marked.*

Silent non – programmable electronic scientific calculators may be used.

Mathematical tables with a list of formulae and squared papers are to be provided.

***All** necessary calculations **must** be shown clearly on the answer sheet provided as the rest of the answers, therefore **no** paper should be provided for rough work.*

*Begin each question on a **fresh** page of the sheet of the answer booklet.*

*Indicate the degree of accuracy at the end of each question attempted using **Cal** for calculator or **Tab** for mathematical tables.*

In numerical work, take acceleration due to gravity g to be $= 9.8\text{ms}^{-2}$.

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SECTION A: (40 marks)

Answer **all** questions in this section.

- Two events A and B are such that $P(A) = 0.4$, $P(B) = 0.7$ and $P(B/A) = 0.3$. Find the probability that neither A nor B occurs. (05 marks)
- Taking 6 ordinates evaluate $\int_{12}^{13} (\sin x) \log x \, dx$, using the trapezium rule to 3 decimal places. (05 marks)
- To a woman in a car travelling at 20kmh^{-1} North East, the wind appears to blow from the West with speed 16kmh^{-1} . Find the actual velocity of the wind. (05 marks)
- The mass of a group of males are normally distributed with mean 80kg and standard deviation 2.6kg . Determine the probability that the mass of a male chosen at random is less than 78.5kg . (05 marks)
- A particle of weight 50N is placed on a smooth plane inclined at arc $\sin\left(\frac{1}{2}\right)$ to the horizontal. Find the horizontal force P , required to keep the body in equilibrium and the normal reaction. (05 marks)
- A random variable Y has probability distribution $f(y)$ as

y	1	2	3	4	5
$P(Y = y)$	0.10	p	0.20	q	0.30

Given that $E(Y) = 3.5$;

Determine;

- the values of p and q ,
 - $P(X \geq 2/X \leq 4)$. (05 marks)
- Show that the equation $e^x = 4 \sin x$ has two roots between $x = 0$ and $x = 1.5$. Hence use linear interpolation once to determine the smaller root correct to 3 decimal places. (05 marks)
 - An object of mass 5kg is initially at rest and is acted upon by a force $\vec{F} = 2\vec{i} + 3\vec{j} - 4\vec{k}$. Find its velocity after 3 seconds. (05 marks)

SECTION B: (60 marks)

Answer any **five** questions from this section.

All questions carry equal marks.

9. (a) A particle is projected from a point O with speed u at an angle θ to the horizontal. Show that its height y above O when it has travelled a distance x horizontally is given by
- $$y = x \tan \theta - \frac{gx^2 \sec^2 \theta}{2u^2}. \quad (04 \text{ marks})$$
- (b) After 2 seconds of projection, a particle projected from the top of a vertical cliff 6 m high with speed $u \text{ ms}^{-1}$ at an angle of elevation θ to the horizontal, passes just above the top of a vertical pole post which is 4 m high and 8 m horizontally away from the base of the cliff.
- (i) Show that $\tan \theta = 2.2$. (05 marks)
- (ii) Find the value of u . (03 marks)

10. Study the table below;

Mass (g)	20 – 24	25 – 34	35 – 38	39 – 44	45 – 49	50 – 54
Frequency density	1.2	1	2	1.5	1	0.4

- (a) (i) Construct a histogram using the data.
- (ii) Use the histogram to estimate the mode.
- (b) Calculate the mean mass. (12 marks)
11. (a) Show that the Newton – Raphson method of finding the root of $\tan x - 4x = 0$ given by
- $$x_{n+1} = \frac{x_n \tan^2 x_n - \tan x_n + x_n}{\tan^2 x_n - 3}$$
- starting with $x_0 = 4.66$, show that $x_2 = 4.6588$.

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- (b) The volume of a bowl of radius, r and depth, h is given by

$$V = \frac{\pi}{3} h^2 (3r - h).$$

Show that the relative error in the volume of a bowl is given by;

$$\frac{\Delta V}{V} = \frac{2 \Delta h}{h} + \frac{3 \Delta r + \Delta h}{3r - h}. \text{ Hence evaluate } \left| \frac{\Delta V}{V} \right| \text{ given that}$$

$$r = 3.21 \text{ cm and } h = 5.5 \text{ cm}.$$

(12 marks)

12. The probability density function of a random variable X is given by;

$$f(x) = \begin{cases} k(x - 2) & ; 2 \leq x \leq 3 \\ k & ; 3 \leq x \leq 5 \\ k(6 - x) & ; 5 \leq x \leq 6 \\ 0 & ; \text{otherwise} \end{cases}$$

Determine;

- (a) the constant k ,
- (b) $P(1.6 \leq x \leq 4.5 / x > 3.2)$,
- (c) the cumulative probability function, $F(x)$.

(12 marks)

13. (a) A multiple choice examination has four alternative answers one of which is right. A student attempted the examination using only guess work. If the examination consisted of ten such questions, calculate the probability that the student got;

- (i) at least four correct answers,
- (ii) more than five correct answers.

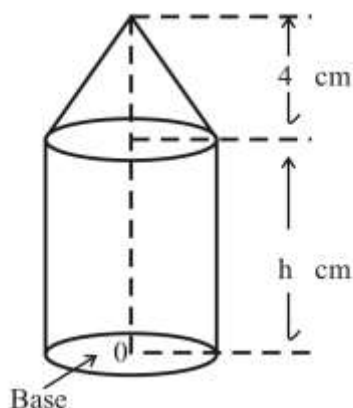
- (b) A machine produces metal pins whose lengths are normally distributed, with mean 2 cm and standard deviation 0.01 cm . The pin is only suitable for its purpose, if its length lies between 0.994 and 1.016 cm .

- (i) Find the probability that a randomly chosen pin is suitable.
- (ii) If the machine is adjusted such that 1.2% of the pins are suitable, find the new standard deviation of the length of the pins, keeping the mean constant.

(12 marks)

14. (a) The figure below shows a uniform solid consisting of a right circular

cone and a cylinder. Show that the distance of the centre of gravity of the solid from the base O is $\frac{3h^2 + 8h + 8}{6h + 8} \text{ cm}$, where h is the height of the cylinder.



- (b) When the solid in (a) above is placed with its base on a rough plane inclined at 45° to the horizontal it will be at the point of toppling. If the radius of the cylinder is $2\frac{2}{3} \text{ cm}$, find the value of h .

(12 marks)

15. The frictional resistance to the motion of a car of mass 1000 kg is kV Newtons where V is its speed and k is a constant. The car ascends a hill of inclination 1 in 10 at a steady speed of 8 ms^{-1} . The power exerted by the engine being 9.76 kW .

Find;

- (a) (i) the value of k ,
(ii) the steady speed at which the car ascends the hill when the power is 12.8 kW .
(b) When the car descends the hill at the speed obtained in (a) (ii) above, the power exerted by the engine is decreased to 1.6 kW . Find the immediate acceleration of the car.

(12 marks)

16. (a) The table below is an extract from table of sines.

θ	$0'$	$6'$	$12'$	$18'$	$24'$	$30'$
$\sin 10^\circ$	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822

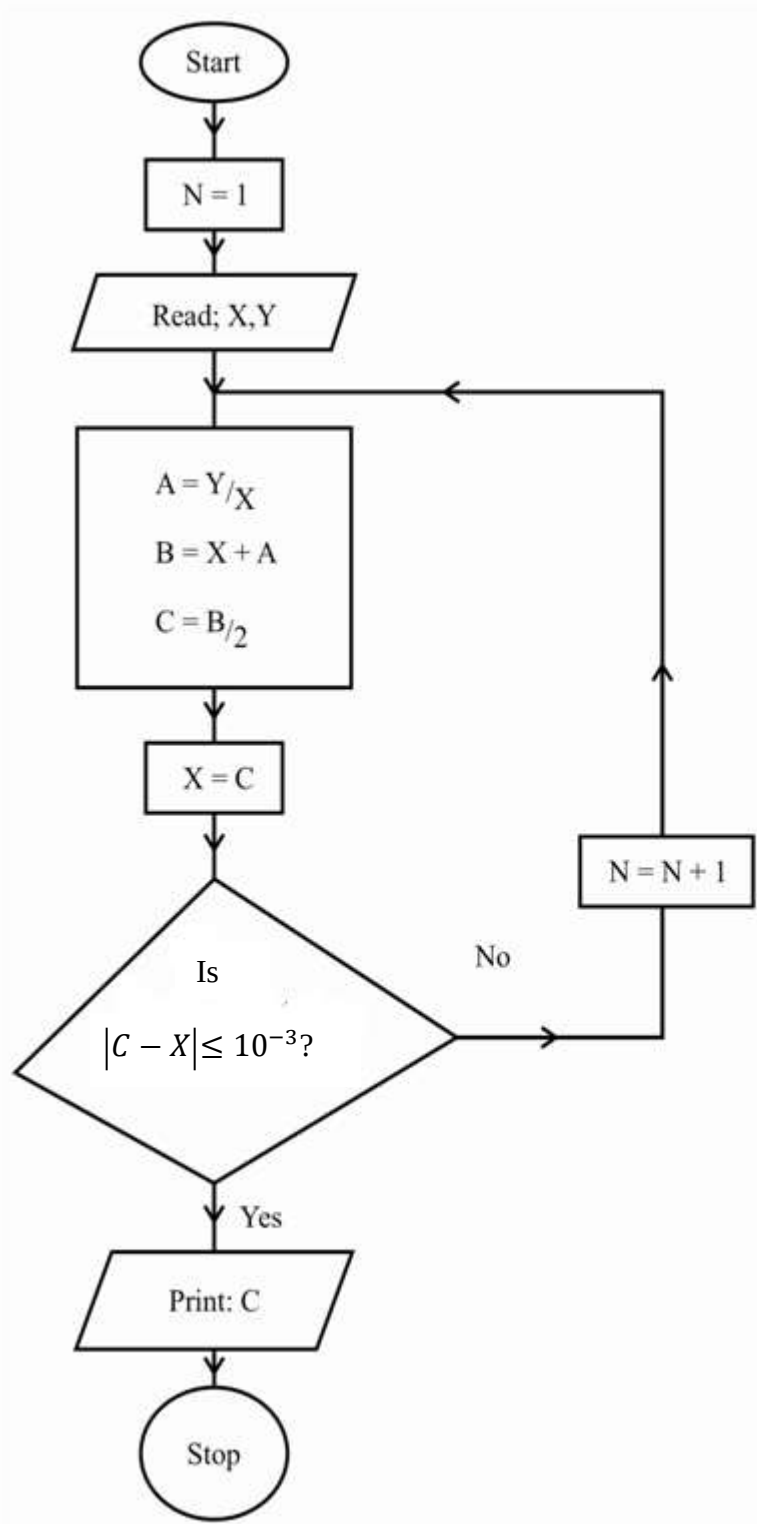
Using linear interpolation or extrapolation, determine the value of:

- (i) $\sin 10^\circ 38'$,
(ii) $\sin^{-1} 0.1747$

(05 marks)

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- (b) Study the flow chart below;



- (i) Perform the dry run of the flow chart for $Y = 60$ and $X = 7$.
- (ii) State the purpose of the flow chart.

(07 marks)

END