P425/1
PURE MATHEMATICS
Paper 1
TUESDAY, 7th August 2018 (Morning)
3 hours

ACHOLI SECONDARY SCHOOLS EXAMINATIONS COMMITTEE

Uganda Advanced Certificate of Education

Joint Mock Examinations, 2018

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- ✓ Answer ALL the EIGHT questions in section A and any FIVE questions in section B.
- ✓ All necessary working MUST be shown clearly.
- ✓ Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 marks)

Answer ALL the questions in this section. All questions carry equal marks.

1. A curve is defined by the parametric equations: $x = t^2$, $y = \frac{1}{t}$ $(t \neq 0)$. Find the equation of the tangent to the curve at the point where the curve cuts the x-axis. (05 marks)

2. If $z = 2 + i$ is a root of the equation $2z^3 - 9z^2 + 14z - 5 = 0$, find the other roots. (05 marks)

3. (i) Find the binomial expression of $\frac{1}{(a + bx)^2}$ up to and including the term x^3.

(ii) Given that the coefficient of the x term is equal to the coefficient of the x^2 term, show that $3b + 2a = 0$. (05 marks)

4. Find the coordinates of the point C on the line joining the points A(–1, 2) and B(–9, 14) which divides AB internally in the ratio 1 : 3. Find also the equation of the line through C which is perpendicular to AB. (05 marks)

5. Solve the equation $5 \text{ Cos } \theta - 3 \text{ Sin } \theta = 4$ for $0^\circ \leq \theta \leq 360^\circ$. (05 marks)

6. Evaluate $\int_2^5 x\sqrt{(x - 1)}\, dx$

(05 marks)

7. Find the Cartesian equation of the plane passing through the midpoint of AB with A(–1, 2, –5) and B(3, 0, –1) which is perpendicular to the line $\frac{x - 1}{2} = \frac{y + 7}{-3} = \frac{6 - z}{8}$. (05 marks)

8. Solve the equation: $\frac{dy}{dx} - y \tan x = \text{Cos } x$, given $y = 0$ at $x = \frac{®}{2}$. (05 marks)

SECTION B (60 marks)

Answer only FIVE questions from this section. All questions carry equal marks.

Question 9:

(a)Evaluate $\displaystyle\int_{-1/2}^{1/2} \frac{(5x^3 - 12x + 4)}{\sqrt{(1 - x^2)}}\,dx$ (05 marks)

(b) By means of the substitution t = tan x, prove that $\displaystyle\int_0^{\text{®}/4} \frac{dx}{1 + \sin 2x} = \frac{1}{2}$ and find the value of $\displaystyle\int_0^{\text{®}/4} \frac{dx}{(1 + \sin 2x)^2}$. (07 marks)

Question 10:

(a) If z_1 and z_2 are complex numbers, solve the simultaneous equations:

$4z_1 + 3z_2 = 23$

$z_1 + iz_2 = 6 + 8i$, giving your answers in the form $x + iy$. (06 marks)

(b) Find the value of the complex number z given that $z^3 = \dfrac{5 + i}{2 + 3i}$. (06 marks)

Question 11:

(a) Find the angle between line $\dfrac{x - 2}{4} = \dfrac{y}{3} = \dfrac{z - 1}{2}$ and the plane $-3x + 5y + 6z = 10$. (04 marks)

(b) A plane P_1 passing through the points (1, –1, 0) and (1, 0, –3) is perpendicular to the plane P_2 having equation: $x + y = 6z = 0$. Find:

(i) the equation of P_1

(ii) the angle between P_1 and another plane P_3 with equation: $x - y + z = 7$.

(08 marks)

Question 12:

A disease is spreading at a rate proportional to the product of the number of people already infected and those who have not yet been infected. Assuming that the total number of people exposed to the disease is N;

(a) Write down a differential equation.

(b) Initially 20% of the population is infected. Two months later 40% of the population is infected. Determine how long it takes for only 25% of the population to remain uninfected. (12 marks)

Question 13:

(a) Determine the equation of the circle passing through the points A(–1, 2), B(2, 4) and C(0,4). (06 marks)

(b) If $y = mx - 5$ is a tangent to the circle $x^2 + y^2 = 9$, find the possible values of m. (06 marks)

Question 14:

Sketch the curve $y = \dfrac{x + 1}{(x - 1)(2x + 1)}$, showing clearly the asymptotes and turning points. (12 marks)

Question 15:

(a) Determine the maximum value of the expression: $6 \sin x - 3 \cos x$. (03 marks)

(b) Prove that $\dfrac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$ (03 marks)

(c) In a triangle ABC, prove that $\sin B + \sin C - \sin A = 4 \cos \dfrac{A}{2} \sin \dfrac{B}{2} \sin \dfrac{C}{2}$ (06 marks)

Question 16:

(a) Using Maclaurin's theorem, expand $e^{-x}\sin x$ up to the term in x^3. Hence, evaluate $e^{-\text{®}/3} \sin \dfrac{\text{®}}{3}$ to 4 significant figures. (05 marks)

(b) The curve $y = x^3 + 8$ cuts the x and y axes at the points A and B respectively. The line AB meets the curve again at point C. Find the coordinates of A, B, C and hence, find the area enclosed between the curve and the line. (07 marks)

THE END