JINJA JOINT EXAMINATIONS BOARD

MOCK EXAMINATIONS 2019

MAKING GUIDE 2019 FOR

P425/1 PAPER 1 MATHEMATICS

SECTION A (40 MARKS)

1. $\cos(45^0 - x) = 2\sin(30^0 + x); -180 \le x \le 180^0$

$\cos 45^0 \cos x - 2\sin 45^0 \sin x = 2\sin 30 \cos x + 2\cos 40^0 \sin x$ M1

$\cos 45^0 \cos x - 2\sin 30^0 \cos x = 2\cos 30^0 \sin x + 2\sin 45^0 \sin x$

$\cos x\,[2\cos 45^0 - 2\sin 30^0] = \sin x\,[2\cos 30^0 + \sin 45^0]$

$\therefore \sin x\,[2\cos 30^0 + \sin 45^0] = \cos x\,[2\cos 45^0 - 2\sin 30^0]$ M1

$\tan x = \frac{\cos 45^0 - 2\sin 30^0}{2cos30^0 + sin45^0}$ M1

$\tan x = -0.1201$

$x = tan^{-1}(-0.1201)$ M1

$x = -6.8^0, 173.2$ A1

05

2. $\frac{2x^2-7x-4}{3x^2-14x+11} > 2$

$$\frac{2x^2-7x-4}{3x^2-14x+11} - 2 > 0$$

$\frac{4x^2+4x+26}{3x^2-14x+11} < 0$ M1

$$\frac{(x-2)(4x-13)}{(x-1)(3x-11)} < 0$$

Critical values ;

$x = 2, x = \frac{13}{4}, x = 4, x = \frac{11}{3}$. B1

	$x < 1$	$1 < x < 2$	$2 < x < \frac{13}{4}$	$\frac{13}{4} < x < \frac{11}{3}$	$x > \frac{11}{3}$
$(x-2)$	$-$	$-$	$+$	$+$	$+$
$(4x-13)$	$-$	$-$	$-$	$+$	$+$
$(x-1)$	$-$	$+$	$+$	$+$	$+$
$(3x-11)$	$-$	$-$	$-$	$-$	$+$
$(x-2)(4x-13)$	$+$	$+$	$-$	$+$	$+$
$(x-1)(3x-11)$	$+$	$-$	$-$	$-$	$+$

$\frac{(x-1)(4x-13)}{(x-1)(3x-11)}$	$+$	$-$	$+$	$-$	$+$

B1

The solution set is $1 < x < 2$ and $\frac{13}{4} < x < \frac{11}{3}$. A1 A1

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3. $\int_0^{\sqrt{\frac{\pi}{2}}} xcox^2 dx$

$\therefore xcosx^2 = \frac{1}{2}\frac{d}{dx}(sinx^2)$ M1

$\Rightarrow \int_0^{\sqrt{\frac{\pi}{2}}} xcox^2 dx = \int_0^{\sqrt{\frac{\pi}{2}}} \frac{1}{2}\frac{d}{dx}(sinx^2)dx$ M1

$= \frac{1}{2} sinx^2 \Big|_0^{\sqrt{\frac{\pi}{2}}}$ M1

$= \frac{1}{2} sin\left(\sqrt{\frac{\pi}{2}}\right)^2 - \frac{1}{2}\sin(0)^2$ M1

$= \frac{1}{2} \times (4)$

$\therefore \int_0^{\sqrt{\frac{\pi}{2}}} x\cos x^2 dx = \frac{1}{2}$ A1

05

4. (i) $x^2 + y^2 - 2x - 8y - 8 = 0$

Let (a, b) be the centre

Comparing:

$x^2 + y^2 + 2gx + 2ty + c = 0$

x;

$2g = -2, \quad \Longrightarrow g = -1$

But;

$a = -g$

$a = -(-1)$
$a = 1$either
y;
2t $= -8$ B1
t $= -4$ or

But
$b = -f$
$b = -(-4)$
$b = 4$ B1
$\therefore$ cetre is the point $(1, 4)$ A1

(ii) Distance between centre and point A

t = $\sqrt{(1+5)^2 + (4+4)^2}$ B1
t = 10 units
shortest distance , d
d = |t – r| M1
d = |10 – 5|
d = 5 units A1

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5. Let x be the number of committees.

B1 B1

$\Longrightarrow$ $x = 3c_3 \times 5c_3 + 3c^2 \times 5c_4$ M1

$x = 10 + 15$

$x = 35$ committees A1

<u>04</u>

6. $cos2x\frac{dy}{dx} = e^x cosex + 3x; \quad y(\pi/2) = 3$

$$\frac{dy}{dx} = e^x + 3xsinx$$

$\int \frac{dy}{dx}dx = \int (e^x\, dx + 3xsinx)dx$

y = $\int e^x dx + 3\int e^x dx + 3\int xsinx\, dx$

y = $e^x + 3\int xsinx\, dx$

$\int x \sin x dx$

$4 = x$, $v = \int \sin x dx$

$\frac{dy}{dx} = 1$ $v = -\cos x$

$\Rightarrow \int x\sin dx = -x\cos x + \int \cos x dx$ M1

$\therefore \int x\sin x dx = -x\cos x + \sin x$

y = $e^x + 3(-xcosx + sinx) + c.$

y = $e^x + -3x\ cosx + 3sinx + c$ B1

when$x = {}^{\pi}/_{2}$, $y = 3$

$\Longrightarrow 3 = e^{\frac{\pi}{2}} - 3 \times \frac{\pi}{2} cos\frac{\pi}{2} + 3sin\frac{\pi}{2} + c$ M1

$3 = e^{\frac{\pi}{2}} + 3 + c$

$c = e^{\frac{\pi}{2}}$ B1

$\therefore y = e^{\frac{\pi}{2}} - 3xcosx + 3sinx$ A1

05

7. Cartesian equation of line: $\frac{x+4}{2} = \frac{2-y}{2} = \frac{Z+3}{4}$, P (0, 6, 0)

$$\therefore \boldsymbol{r} = \begin{pmatrix} -4 \\ 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$$

$\Longrightarrow \overrightarrow{MP} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} -4+2t \\ 2-2t \\ 3-4t \end{pmatrix}$ B1

$\overrightarrow{MP} = \begin{pmatrix} -4+2t \\ 2-2t \\ 3-4t \end{pmatrix}$

But $\overrightarrow{MP}\boldsymbol{b} \bullet = 0$

$\begin{pmatrix} -4+2t \\ 2-2t \\ 3-4t \end{pmatrix}\begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} \bullet = 0$ M1

$2(4+2t) + -2(-2+2t) + 4(3-4t) = 0$

8 – 4t + 4 + 4t + 12 – 12t = 0

12 + 12 – 12t = 0

−12t = −24

t = $\frac{-24}{-12}$ B1

t = 2.

$\therefore \overrightarrow{MP} = \begin{pmatrix} 0 \\ 2 \\ -5 \end{pmatrix}$

Distance of point C (0, 6, 0) from the line.

$\Longrightarrow \quad |\overrightarrow{MP}| = \sqrt{(0)^2 + (2)^2 + (-5)^2}$ M1

$|\overrightarrow{MP}| = \sqrt{29}$ units. A1

05

8. $x = 1 + \cos 2\theta \quad y = \sin\theta$

$x = 2\cos^2\theta$

$\frac{dx}{d\theta} = -4\cos\theta\sin\theta \quad \frac{dy}{d\theta} = \cos\theta$ M1 M1

Using:

$$\frac{dy}{dx} = \frac{dy}{d\theta} : \frac{d\theta}{dx}$$

$$= \cos\theta \times \frac{-1}{4\cos\theta\sin\theta}$$

$$\frac{dy}{dx} = \frac{-1}{4\sin\theta}$$ B1

But $\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{-1}{4\sin\theta}\right) \times \frac{1}{-4\sin\theta\cos\theta}$$

$$= \frac{1}{16}(-\operatorname{cosec}\theta\cot\theta)\frac{1}{\sin\theta\cos\theta}$$ M1

$$= \frac{-1}{16}\left(\frac{1}{\sin\theta}\right)^3$$

$= 4\left(\frac{dy}{dx}\right)^3$ B1

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SECTION B

9. (a) $x - 10y + 7z = 13$ ——— (i)

$x + 4y - 3z = -3$ ——— (ii)

$-x + 2y - z = -3$ ——— (iii)

Method: Elimination

(i) ——— (ii)

$-14y + 6z = 16$

$7y - 5z = -8$ ——— (iv) M1

(i) + (ii)

$-8y + 6z = 16$ M1

$4y + 3z = -5$ ——— (v)

3 (iv) ——— 5(v)

$y = +1$

From (iv):

$\Rightarrow 7y - 5z = -8$

$\therefore 7(1) - 5z = -8$ M1

$-z = 3$

From (i)

$x - 10y + 7z = 13$

$x - 10(1) + 7(8) = 13$ M1

$x = 2$

$x = 13 - 11$

$x = 2$

A1 A1A1

$\therefore x = 2,\ y = 1,\ z = 3$

(b) $P(x) = ?\ g(x) = ?\ f(x) = x^2 - 5x - 14$

Using:

$P(x) = g(x) f(x) + R(x)$

But $R(x) = 2x + 5.$

$\Rightarrow P(x) = g(x)(x + 2x)(x - 7) + 2x + 5$ M1

(i) Let $x = 7.$

$P(7) = g(7)(7 \ + 2)(7 - 7) + 2 \times 7 + 5$ M1

$P(7) = 14 + 5$

$P(7) = 19$

$\therefore$ The remainder is 19. A1

(ii) Let $x = -2$

$\Rightarrow P(-2) = g(-2)(-2 \ + 2)(-2 - 7) + 2 \times 2 + 5$ M1

$P(-2) = -4 + 5$

$P(-2) = 1$

$\therefore$ The remainder is 1. A1

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10. (a) $4\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$

$4\sin\theta - 3\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

Compering:

$\sin\theta$;

$R\sin\alpha = 4$ ———————— (i)

$\cos\theta$;

$R\cos\alpha = 3$ ———————— (ii)

Value of R

(i)^2 + (ii)^2

$(R\sin\alpha)^2 + (R\cos\alpha)^2 = (4)^2 + (3)^2$ M1

$R^2\left[\sin^2\alpha + \cos^2\alpha\right] = 16 + 9$

$R^2 = 25$

$\therefore R = 5$ B1

Size of angle, α

(i) $\div$ (ii)

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{4}{3}$

$\tan\alpha = \frac{4}{3}$

$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$ B1

$\alpha = 53.1°$

$\therefore\ 4\sin\theta - 3\cos\theta = 5\sin(\theta - 53.1)$

Solving the equation $4\sin\theta - 3\cos\theta + 2 = 0$

$\Rightarrow 5\sin(\theta - 53.1^0) + 2 = 0$ M1

$\sin(\theta - 53.1^0) = \frac{-2}{5}$

$\theta - 53.1^0) = \sin^{-1}\left(\frac{-2}{5}\right)$ M1

$\theta - 53.1^0 = 203.6°, 336.4°$

$\theta = 256.7°, 389.5°$

$\therefore \theta = 256.7°$ A1

(b)

From the sine rule

LHS

$\Rightarrow \frac{a+b-c}{a-b+c} = \frac{2R\sin A + 2R\sin B - 2R\sin C}{2R\sin A + 2R\sin B + 2R\sin C}$ M1

$= \frac{\sin A + \sin B - \sin C}{\sin A - \sin B + \sin C}$

$= \frac{2\sin\frac{A}{2}COS\frac{A}{2} + 2COS\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2} - 2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}$ M1

$$= \frac{\sin\frac{A}{2}COS\frac{A}{2} + COS\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}\cos\frac{A}{2} - \cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}$$

But A + B + C = 180^0

A = 180 – (B + C)

$$\frac{A}{2} = 90^0 - \left(\frac{B+C}{2}\right)$$

$$\sin\frac{A}{2} = \sin\left[90^0 - \left(\frac{B+C}{2}\right)\right]$$

$$\sin\frac{A}{2} = \cos\left(\frac{B+C}{2}\right)$$ B1

Also;

$$\cos\frac{A}{2} = \cos\left[90 - \left(\frac{B+C}{2}\right)\right]$$

$$\cos\frac{A}{2} = \sin\left(\frac{B+C}{2}\right)$$ B1

$$\Rightarrow \frac{a+b-c}{a-b+c} = \frac{\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B+C}{2}\right) + \cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B+C}{2}\right) - \cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}$$

$$= \frac{\sin\left(\frac{B+C}{2}\right) + \sin\left(\frac{B-C}{2}\right)}{\sin\left(\frac{B+C}{2}\right) - \sin\left(\frac{B-C}{2}\right)}$$

$$= \frac{2\sin\frac{B}{2}\cos\frac{C}{2}}{2\cos\frac{B}{2}\sin\frac{C}{2}}$$ M1

$$\therefore \frac{a+b-c}{a-b+c} = \tan\frac{B}{2}\tan\frac{C}{2}$$ B1

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11.

$y^2 = 4ax$

$2y\frac{dy}{dx} = 4a$ M1

$2\frac{dy}{dx} = \frac{2a}{y}$

At the point $P(at^2, 2at)$:

$\Rightarrow \frac{dy}{dx} = \frac{2a}{2at}$

$\frac{dy}{dx} = \frac{1}{t}$

Using: $m_1 \times m_2 = -1$ M1

$\Rightarrow \frac{1}{t} \times m_2 = -1$

$m_2 = -t$ B1

Equation of normal at the point $P(at^2, 2at)$

$\frac{y - 2at}{x - at^2} = -t$ M1

$y - 2at = -t(x - at^2)$

$y = -tx + at^2 + 2at$

$y = -tx + at(t^2 + 2)$

Coordinates of the point G

x - intercept occurs when y = 0.

$\Rightarrow 0 = -tx + at(t^2 + 2)$ M1

$x = a(t^2 + 2)$

G is the point

y – coordinate of P $\left[a(t^2 + 2), 0\right]$ B1

Let Q be the point (x, y).

P is the midpoint of G and Q.

x - coordinate of P.

$\Rightarrow at^2 = \frac{1}{2}(x + a(t^2 + 2)$ M1

$2at^2 = x + a(t^2 + 2)$

$2at^2 - +at^2 - 22 = x$

$\therefore x = a(t^2 - 2)$ ———————— (i) B1

$$\Rightarrow 2at = \frac{y+0}{2}$$ M1

$$t = \frac{y}{4a} \text{ ———————————— (ii)}$$ B1

Substitute (ii) in (i) for t

$$\Rightarrow x = a\left[\left(\frac{y}{4a}\right)^2 - 2\right]$$ M1

$$\therefore y^2 = 16a(x+2a)$$ B1

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12. (i)

$$Z_1 = \frac{1+i\sqrt{3}}{2}$$

$$r_1 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$ B1

$$r_1 = 1\text{unit}$$

Also;

$$\theta_1 = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$$

$$\theta_1 = \tan^{-1}\left(\sqrt{3}\right)$$

$$\theta_1 = \frac{\pi}{3}$$ B1

$$\Rightarrow Z_1 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$$

$$Z_2 = \frac{1-i\sqrt{3}}{2}$$

$$r_2 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$$ B1

$$r_2 = 1\text{unit}$$

$$Q_2 = \tan^{-1}\left(\frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}}\right)$$

$$Q_2 = \tan^{-1}\left(-\sqrt{3}\right)$$

$$Q_2 = \frac{-\pi}{3} \, or \, \frac{2\pi}{3}$$ B1

$$\Rightarrow Z_2 = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$$

Or

$$\Rightarrow Z_1 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$ A1

(ii) $$Z_1^5 + Z_2^5 = \left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]^5 + \left[\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right]^5$$ M1

$$= \cos 5\frac{\pi}{3} + i\sin 5\frac{\pi}{3} + \cos 5\frac{\pi}{3} - i\sin 5\frac{\pi}{3}$$

$$= 2 \times \cos 5\frac{\pi}{3}$$

$$= 2 \times \frac{1}{2}$$

$$Z_1^5 + Z_2^5 = 1$$ A1

(b) $Z_1 = -4 - 3i, \Rightarrow Z_2 = -4 + 3i$ is also a root. A1

Using:

$$Z^2 - (-4 - 3i + -4 + 3i)Z + (-4 - 3i)(-4 + 3i) = 0$$

$$Z^2 + 8Z + 25 = 0$$

$\therefore Z^2 + 8Z + 25 = 0$ is a quadratic factor.

Solving for the roots.

$$\begin{array}{r|l} & Z^2 - 12Z + 37 \\ \hline Z^2 + 8Z + 25 & Z^4 - 4Z^3 - 34Z^2 - 4Z + 925 \\ & -(Z^4 + 8Z^3 + 25Z^2) \\ & -12Z^3 - 59Z^2 - 4Z + 925 \\ & -(-12Z^3 - 96Z^2 - 300Z) \\ & 37Z^2 + 296Z + 925 \\ & -(37Z^2 + 296Z + 925) \\ & 0 \end{array}$$ M1

Solving;

$Z^2 - 12Z + 37 = 0$

$Z = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 1 \times 37}}{2(1)}$ M1

$Z = \frac{12 \pm \sqrt{(144 - 148}}{2}$

$Z = \frac{12 \pm 2i}{2}$

$Z = 6 \pm i$

$\therefore$ Other roots are; $-4-3i,\ 6+i$ and $6-i$ A1 A1

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13.

Let $\frac{5x^2 - 8x + 1}{2x(x-1)^2} = \frac{A}{2x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$ M1

$5x^2 - 8x + 1 = A(x-1)^2 + B \times 2x(x-1) + C2x$

Let $x = 1$

$\Rightarrow 5(1)^2 - 8(1) + 1 = C \times 2(1)$ M1

$\Rightarrow C = -1$ B1

Let $x = 0$ M1

$\Rightarrow 5(0)^2 - 8(0) + 1 = A(0-1)^2$ B1

$\Rightarrow A = 1$

Coefficient of x^2 :

$5 = A + 2B$ M1

$5 = 1 + 2B$

$4 = 2B,\ B = 2$ B1

$\Rightarrow \int_4^9 \frac{5x^2 - 8x + 1}{2x(x-1)^2} dx = \int_4^9 \frac{dx}{2x} + \int_4^9 \frac{2dx}{(x-1)} + \int_4^9 \frac{-dx}{(x-1)^2}$ M1 B1

$= \frac{1}{2} Inx\Big|_4^9 + 2In(x-1)\Big|_4^9 + \frac{1}{(x-1)}\Big|_4^9$ M1

$= \left[In\sqrt{2 \times 9} - In\sqrt{2 \times 4}\right] + \left[In(9-1)^2 - In(4-1)^2\right] + \left[\frac{1}{9-1} - \frac{1}{(4-1)}\right]$

M1

$\therefore \int_4^9 \frac{5x^2 - 8x + 1}{2x(x-1)^2} dx = In\left(\frac{32}{3}\right) - \frac{5}{24}$ B1

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14. (a) $\overrightarrow{OA} = 3i - j + 2k$

$\overrightarrow{OB} = -\mathbf{i} + \mathbf{j} + 9\mathbf{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (-1-3)i + (1-^{-}1) + (9-2)\mathbf{k}$ B1

$\overrightarrow{AB} = -4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$

Using:

$\mathbf{r} = \overrightarrow{OA} + \mu\overrightarrow{AB}$

$\mathbf{r} = (3\mathbf{i} - \mathbf{j} + 2k + \mu(-4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$ M1 A1

Or

$$\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \mu\begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix}$$

(b) line L_1:

$$\mathbf{r}_1 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \mu\begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix}$$ B1

Line L_2 :

$$\mathbf{r}_2 = \begin{pmatrix} 8 \\ 1 \\ -6 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$$ B1

At the point of intersection

$\mathbf{r}_1 = \mathbf{r}_2$

$$\Rightarrow \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \mu\begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ -6 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$$ M1

$3 - 4\mu = 8 + \lambda, \Rightarrow 4\mu + \lambda = -5$ ———— (i) B1

$-1 + 2\mu = 1 - 2\lambda, \Rightarrow \mu + \lambda = 1$ ———— (ii) B1

$2 + 7\mu = -6 - +2\lambda, \Rightarrow 7\mu + 2\lambda = -8$ ———— (iii) B1

Solving (i) and (ii)

(i)————(ii) M1

$3\mu = -6$

$\mu = -2$

From (i)

$\mu + \lambda = 1$ M1

$2 + \lambda = 1$

$\lambda = 3$

From:

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + -2 \begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ -6 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 11 \\ -5 \\ -12 \end{pmatrix} = \begin{pmatrix} 11 \\ -5 \\ -12 \end{pmatrix}$$

$\therefore$ The lines intersect. B1

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15. $y = \dfrac{x^2 - 2x + 1}{x^2 - 2x - 3} = 1 + \dfrac{-x + 3}{(x+1)(x-2)}$

(a) (i) Horizontal asymptote.

A1 A1

$\therefore y = 1$ is a horizontal asymptote and $x = -1, x = 2$ are vertical asymptote

Vertical asymptote

For stationary points,

(ii) $\dfrac{dy}{dx} = \dfrac{(2x-2)(x^2 - x - 2) - (2x-1)(x^2 - 2x + 1)}{(x^2 - x - 2)^2} = 0$ B1

$x^2 - 6x + 5 = 0$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times 5}}{2(I)}$

$x = 5$ or $x = 1$

Stationary points are;

(1, 0) and $\left(5, \dfrac{8}{9}\right)$ A1 A1

Nature of turning point.

$$\frac{dy}{dx} = \frac{x^2 - 6x + 5}{(x^2 - x - 2)^2}$$

x	L	I	R	L	S	R
Sign of $\frac{dy}{dx}$	+	0	–	–	0	+

Max   Min

$\therefore$ Point (1, 0) is a maximum and $(5, {}^{8}/_{9})$ is a minimum. B1

B1

(b) Intercepts of the curve and axes

x - intercepts occurs for y = 0, $x = 1$ either B1

or

y – intercept occurs when $x = 0$, $y = \frac{-1}{2}$

Now As $x \to +\infty$, $y \to 1^-$

As $x \to -\infty$, $y \to 1^+$

$\therefore$ y = 1 is a horizontal asymptote.

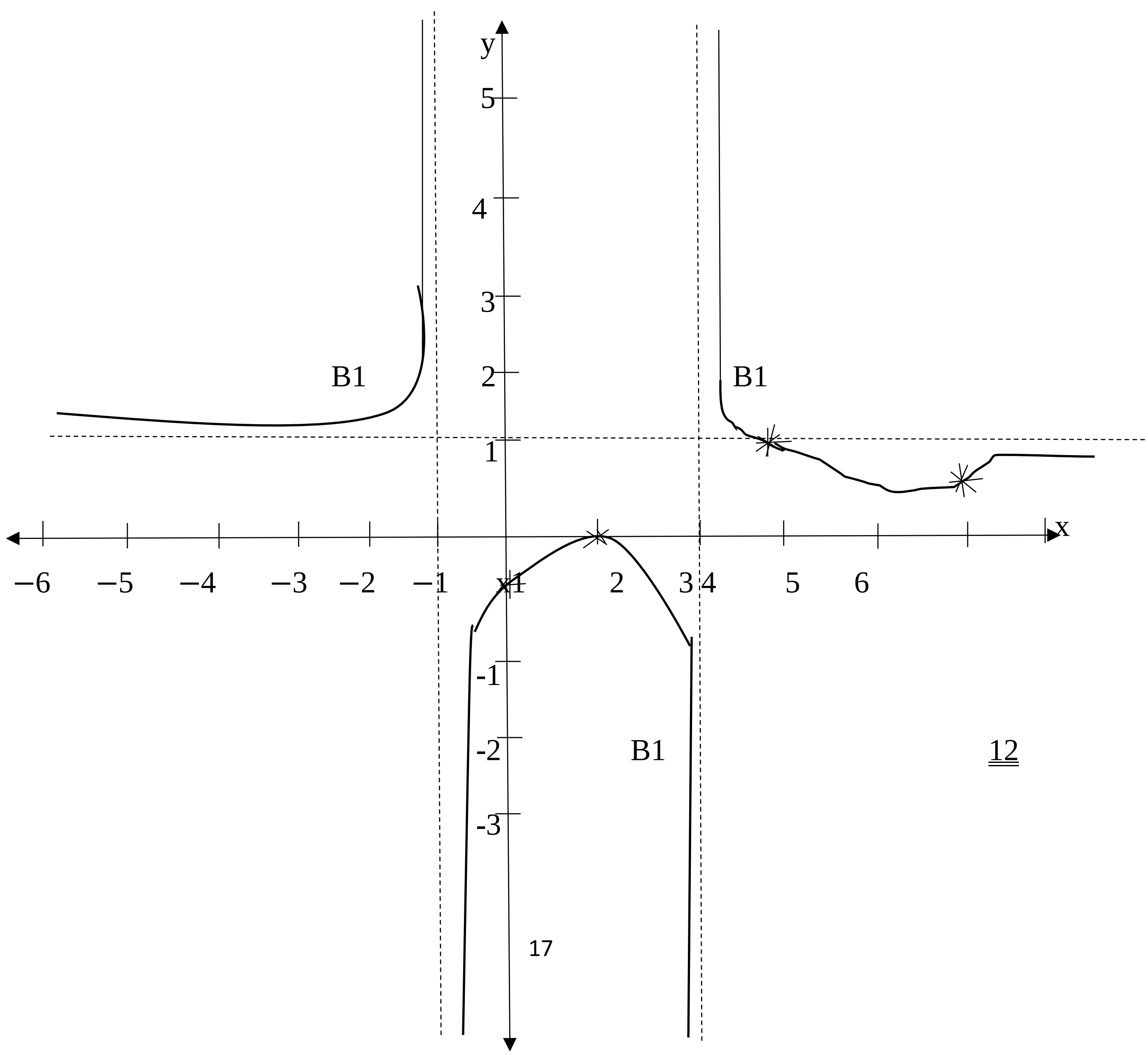
Intercept of curve and the line y = 1

$$\Rightarrow 1 = \frac{x^2 - 2x + 1}{x^2 - x - 2}$$

$$x^2 - x - 2 = x^2 - 2x + 1$$
$$x = 3$$

Point (3, 1) B1

Sketch of the curve.



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$-\overline{4}$

$x=1 \; x=2$

16. (a)

$$\frac{dy}{dx}=\frac{f(x+\delta x)-f(x)}{\delta x}$$ M1

$$=\frac{\frac{1}{(x+\delta x)^2}-\frac{1}{x^2}}{dx}$$ M1

$$=\frac{x^2-(x+\delta x)^2}{x^2(x+\delta x)^2\delta x}$$ B1

$$=\frac{2x\delta x+(fx)^2}{x^2(x+\delta x)^2\delta x}$$

$$=\frac{-2x+(fx)}{x^2(x+\delta x)^2}$$ B1

As $\delta x\to 0, \frac{\delta y}{\delta x}\to\frac{dy}{dx}$

B1 A1

$$\therefore \frac{dy}{dx}=\frac{-2x}{x^2(x)^2}=\frac{-2}{x^2}$$

(b)

$$e^x=\cos)x-y)$$ M1

$$e^x=\left(1-\frac{dy}{dx}\sin(x-y)\right)$$

$$e^x=\sin(x-y)-\sin(x-y)\frac{dy}{dx}$$ M1

$$e^x=\sin(x-y)-\sin(x-y)\frac{dy}{dx}$$

$$\therefore \sin(x-y)\frac{dy}{dx}=\sin(x-y)-e^x$$

$$\frac{dy}{dx} = \frac{\sin(x-y) - e^x}{\sin(x-y)}$$ B1

Recall that:

$$\cos^2(x-y) + \sin^2(x-y) = 1$$

$$\sin(x-y) = \sqrt{1-\cos^2(x-y)}$$ B1

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-\cos^2(x-y)} - e^x}{\sqrt{1-\cos^2(x-y)}}$$

$$\frac{dy}{dx} = \frac{\sqrt{1-(e^x)^2} - e^x}{\sqrt{1-(e^x)^2}}$$ M1

$$\therefore \frac{dy}{dx} = \frac{\sqrt{1-e^{2x}} - e^x}{\sqrt{1-e^{2x}}}$$ B1

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